Blue Pelican

Calculator Appendices

Version 1.01

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Graphing Calculator Appendices

Appendix A: Special marking of graphed functions (video time = 3:27)
Comparing lines (2-02)
Comparing parabolas (8-04, 8-06)

Appendix B: Graphing vertical lines (3-03) (video time = 4:45)

Appendix C: Finding intersection points (2-06) (video time = 3:31)

Appendix D: Graphical solutions to absolute value problems (1-06) (video time = 4:02)

Appendix E: Graphing inequalities in two variables (3-03) (video time = 3:17)

Appendix F: Restricting the domain of functions (7-02) (video time = 5:40)

Appendix G: Even and odd functions (7-04) (video time =3.02)

Appendix H: Inverse functions (9-04) (video time = 4:38)

Appendix I: Finding roots (8-06) (video time = 3:45)

Appendix J: Finding maximum and minimum points (8-06) (video time = 3:22)

Appendix K: Calculations with the natural number e (10-02) (videotime = 3:53)

Appendix L: Logarithms (video time = 4:11)
Common and natural logs (11-05)
Change of base (11-05)

Appendix M: Scatter plots (13-01, 02; 15-06) (video time = 3:16)

Appendix N : Regression (13-01, 02) (video time =6:27)

Appendix O : Factorials, permutations, and combinations (15-01, 02, 03)
(video time = 4:40)

Appendix P: Statistics (15-07)

Appendix Q: Matrix fundamentals (elementary matrix math) (18-01)

Appendix R: Advanced matrix math: mult, inverse, determinant (18-02, 03, 04)
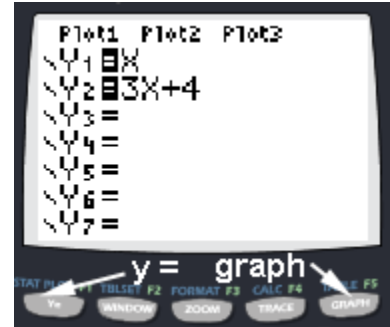
Calculator Appendix A

# (Special marking of graphed functions)

Multiple functions can be simultaneously graphed by pressing the **Y=** button. Enter the first function beside **Y1=** and others beside **Y2=**, etc.

For the sake of illustration, the functions
y = x and y = 3x + 4 are shown here.

After entering the functions, graph them by pressing the **GRAPH** button.

The two graphed functions will appear as shown here. (If not, press **ZOOM | 6: ZStandard** … the zoom button is the middle button in the picture above.)
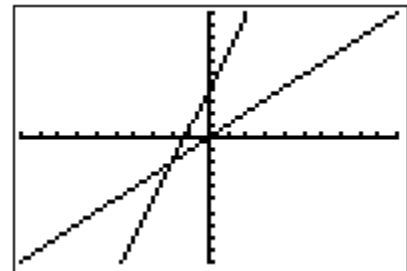
Although it is easy to tell which graph belongs to which function in this particular example, sometimes it is not this easy. This is especially true when there are several functions intertwined or when deep zooms allow only tiny portions to be viewed.

In the present example we will show how to **specially mark one of the functions** so there will be **no doubt concerning its identity** on the graph screen.

We will mark the steeper of the two lines (**Y2** = 3x + 4) by moving with the **UP** and/or **DOWN ARROWS** until the cursor is on the **Y2** function.

At this point we want to move the cursor to the left of **Y2**. Do this by pressing the **LEFT ARROW** twice. Press **ENTER** and the icon to the left of **Y2** will turn into a heavy mark. (Repeated pressing of **ENTER** will toggle through other options.)
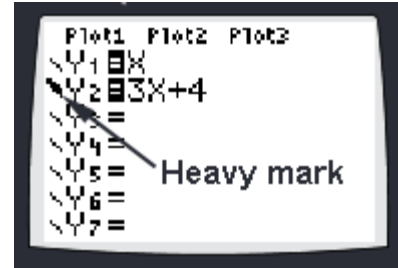
Now press the **GRAPH** button and the **Y2 function** will be observed to be displayed with a heavier graph so as to associate it with the proper function.
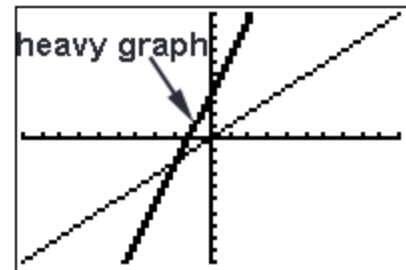
Calculator Appendix B

(Graphing vertical lines)

Generally it is not possible to graph a perfectly vertical line on a TI 84 graphing calculator and have it behave as an ordinary function in the sense that it is possible to find its intersection with other functions.

However, it is possible to create a line so steep that when viewed at a particular zoom level, it essentially looks and behaves as a vertical line.

As an example, let's suppose we need the vertical line x = 5. This cannot directly be entered using the **Y=** button because that particular screen designates functions with **Y1**, **Y2**, etc. (not **X1**, **X2**, etc.)

For the particular zoom area you intend to use, pick a *y* value that is approximately in the middle of the zoom box (let's call it y_{mid}). Use the point-slope form for a line and with the point (5, y_{mid}) and a very large, negative slope of m = 999999999999. Thus, the point-slope equation of this line becomes:

$$\textbf{Y1} - y_{mid} = -999999999999(x - 5)$$

$$\textbf{Y1} = -999999999999(x - 5) + y_{mid}$$

Most zooms are such that the x-axis is visible in the zoom area, so a safe value of y_{mid} to use is 0. In that case our equation would become:
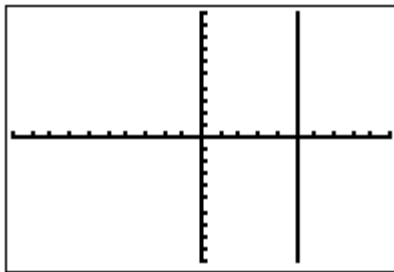
$$\textbf{Y1} = -999999999999(x - 5) + 0$$

$$\textbf{Y1} = -999999999999(x - 5)$$

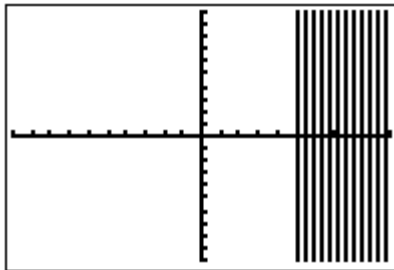For a zoom given by **ZOOM | 6: ZStandard** the display would appear as shown here.

Essentially the same result can be achieved with a positive slope of m= 999999999999; however, the **negative slope must be used** to obtain proper left/right shading when graphing the inequalities x < 5 or x > 5.
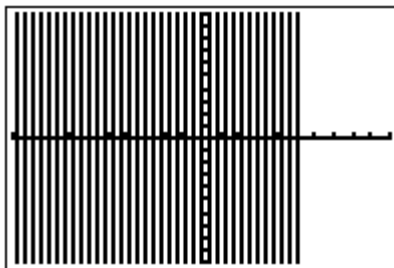
Notice the greater-than ⊿ and less-than ⊾ icons just to the left of **Y1**. These are set by moving the cursor to the left of **Y1** with the **LEFT ARROW** key. Toggle through the various option by pressing **ENTER**.
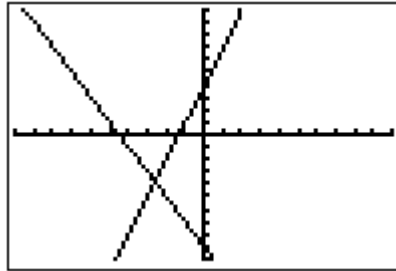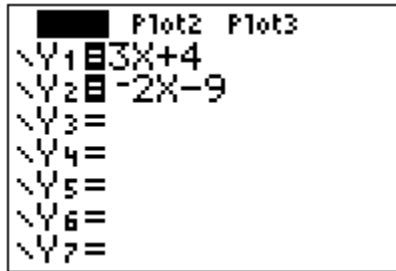
Calculator Appendix C

(Finding intersection points)

Consider finding the intersection point of two functions (y = 3x + 4 and y = -2x – 9). With **Y=**, enter the two functions and press **GRAPH**.

To initiate the process of finding the intersection, press **2ⁿᵈ | CALC**. From the resulting menu choose **5: intersect** by pressing the **DOWN ARROW** button thus moving the cursor all the way down to **5:** and then pressing **ENTER**, or by pressing the **5** button.
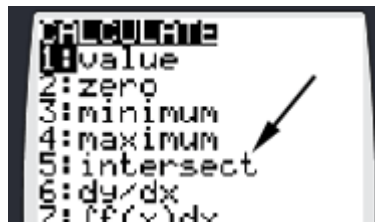
The next screen will show the graph along with a question, "First curve?". It is asking for either one of the curves. A blinker will be positioned on one of the "curves" (actually a line in this example) that is coincidentally also on the y-axis in this example.
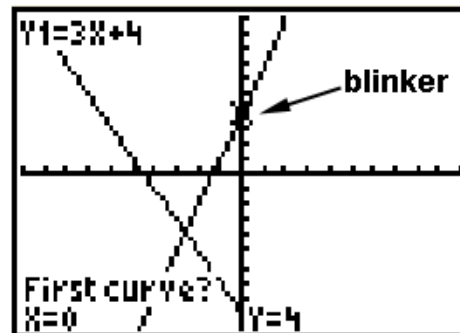
The blinker can be moved along the present curve with the **LEFT/RIGHT ARROWS** (The **UP/DOWN ARROWS** jumps between curves). When satisfied that the blinker is distinctly on one of the curves, press **ENTER**.

At this point another question will appear asking for the second curve.
Again, move the blinker with the **LEFT/RIGHT ARROWS** until satisfied that the blinker is distinctly on the other curve. Press **ENTER**.
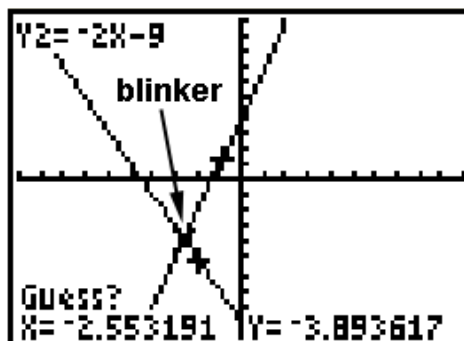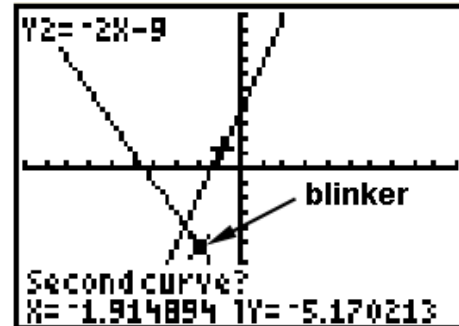
Finally, a third question is presented asking for a, "Guess?" Using the **LEFT/RIGHT ARROWS**, position the blinker approximately over the intersection point and press **ENTER**.

Students often ask why a guess is necessary. It's because the curves are often more complicated than our simple example and **intersect in more than one point**. The calculator needs to know which point we want, so the software inside the calculator insists on a "guess" in all cases.

The coordinates of the point of intersection are now shown at the bottom of the graph screen.

For simple cases like that depeicted here in which there is only one intersection point, it is possible to get the answer with three quick **ENTERS**'s after choosing **5: intersect**. (No need to adjust positions with the **LEFT/RIGHT ARROWS**.)
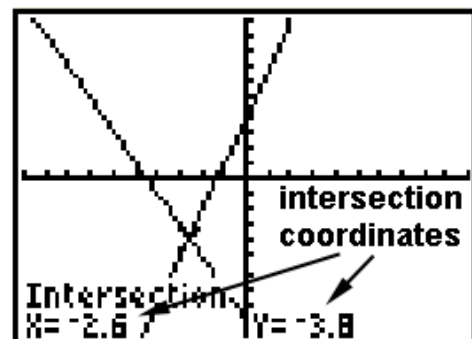
Calculator Appendix D
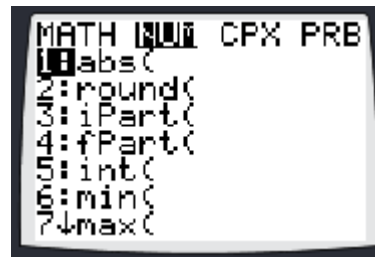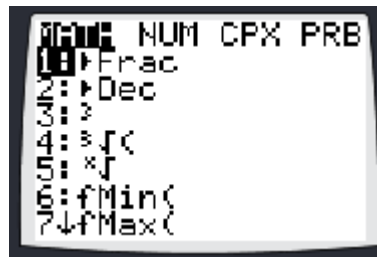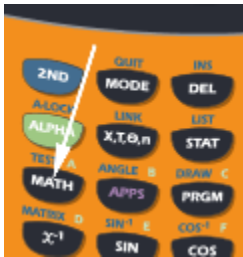
(Graphical solutions to absolute value problems)

Consider the solution to the problem $|x - 4| = 5$. To solve this on a graphing calculator press **Y=** and enter:
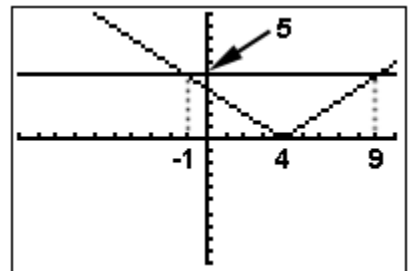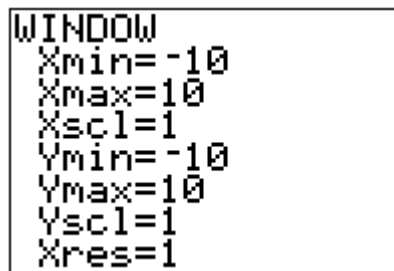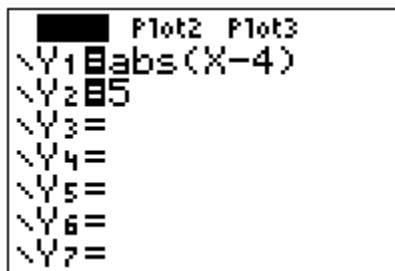
Y1 = abs(x–4)
Y2 = 5

Access the absolute value function (given by **abs** above) with:
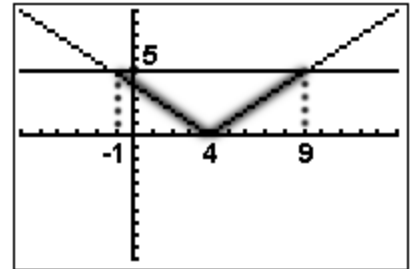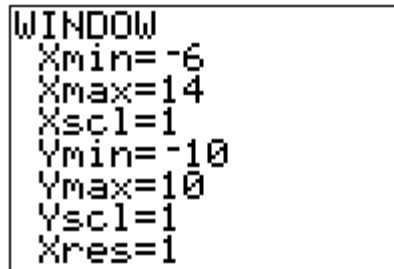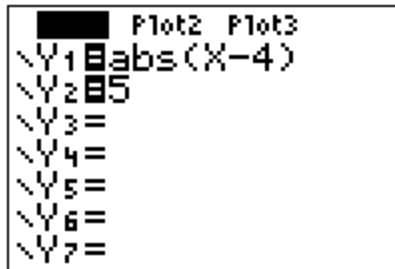**Math | Num | 1: abs(**

The x-coordinates of the two intersection points of these two functions are the solutions to this problem. (See Appendix C for how to find intersection points.)

From the graph below, we see that the solutions are x = -1 and x = 9.

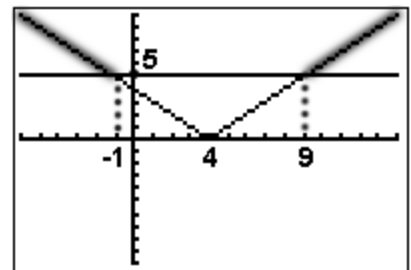Now consider the same absolute value quantity, but this time as an inequality:

$$|x-4| \leq 5$$

The "heavy" part of the graph above is the area in which the **Y1** function is less than (**below**) the **Y2** function. The x values that correspond to this constitute the solution:

-1 ≤ x ≤ 9   which is equivalent to x ≥ -1 and x ≤ 9

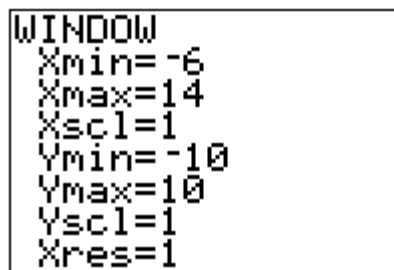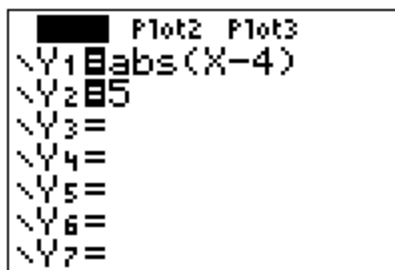Finally, consider the inequality:

$$|x-4| \geq 5$$

The "heavy" part of the graph above is the area in which the **Y1** function is less than (**above**) the **Y2** function. The x values that correspond to this constitute the solution:

x ≤ -1 or x ≥ 9

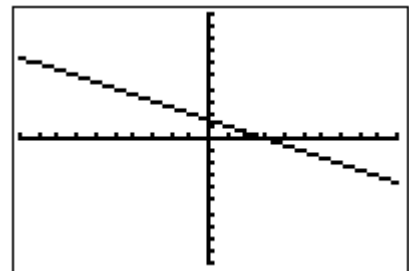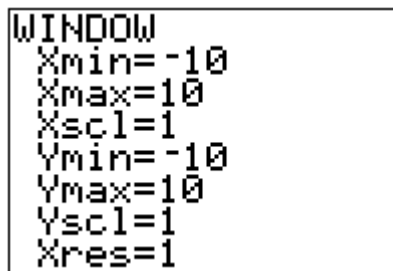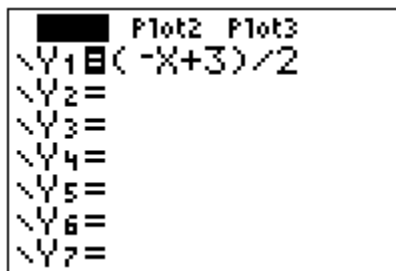# Calculator Appendix E

(Graphing inequalities in two variables)

Consider the display of the following inequality on a graphing calculator.

x + 2y ≥ 3

The first step is to solve for y:

y ≥ (-x + 3)/2

Press the **Y=** button on the calculator and enter (-x + 3)/2 for **Y1**. Press **GRAPH** and observe that a **line** is graphed:

Plot2 Plot3	WINDOW	
\Y1█(-X+3)/2	Xmin=-10	
\Y2=	Xmax=10	
\Y3=	Xscl=1	
\Y4=	Ymin=-10	
\Y5=	Ymax=10	
\Y6=	Yscl=1	
\Y7=	Xres=1	

In order to turn this into a true inequality with a shaded region, use the **LEFT ARROW** to move the cursor to the left of **Y1 =**. Once the cursor is in that position, repeatedly press **ENTER** until this icon (◥) appears. This causes the function (in this case a line) to be shaded **above**. Press **GRAPH** again and shading will be observed **above** the line.

Plot2 Plot3	WINDOW	
◥Y1█(-X+3)/2	Xmin=-10	
\Y2= ←Notice	Xmax=10	
\Y3=	Xscl=1	
\Y4=	Ymin=-10	
\Y5=	Ymax=10	
\Y6=	Yscl=1	
\Y7=	Xres=1	
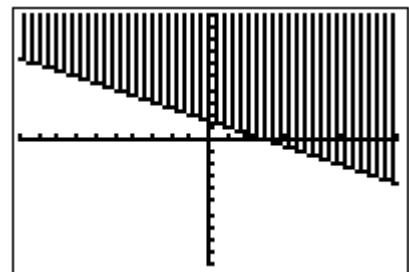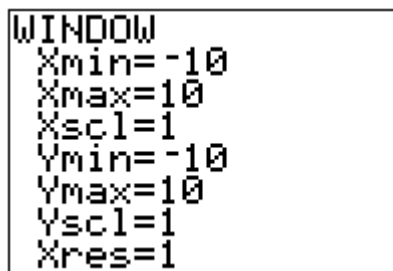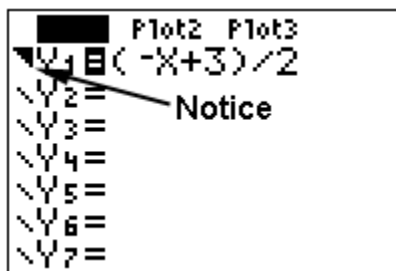
To graph the inequality y ≤ (-x + 3)/2 which will require shading **below** the line, use the **LEFT ARROW** to move the cursor to the left of **Y1 =**. Once the cursor is in that position, repeatedly press **ENTER** until this icon (▸) appears. Press **GRAPH** and shading will be observed **below** the line.

Plot2 Plot3	WINDOW	
▸Y1◼(-X+3)/2	Xmin=-10	
\Y2=	Xmax=10	
\Y3=	Xscl=1	
\Y4=	Ymin=-10	
\Y5=	Ymax=10	
\Y6=	Yscl=1	
\Y7=	Xres=1	
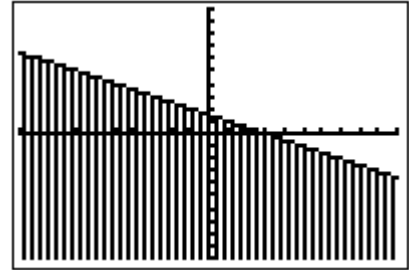
Similarly, the icons ▾ and ▸ will result in **any function** (not just lines) being shaded respectively above or below the graph.

Think of it this way:

   With ▾, **Y1=** … effectively becomes **Y1≥** …, and

   With ▸, **Y1=** … effectively becomes **Y1≤** …

Calculator Appendix F

(Restricting the domain of a function)

It is possible to restrict the domain of a graphed function on a graphing calculator so as to make only a portion of the graph visible.

We will consider the function $f(x) = -.2x^2 + 7$. Press **Y=**, enter the function as **Y1**, and then press **GRAPH**.
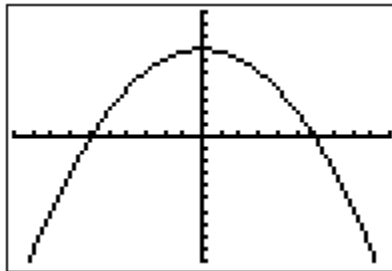
To restrict the domain to **x > 5**, append **/(x > 5)** to the end of the function specified by **Y1**. Be sure to enclose the function in parenthesis.
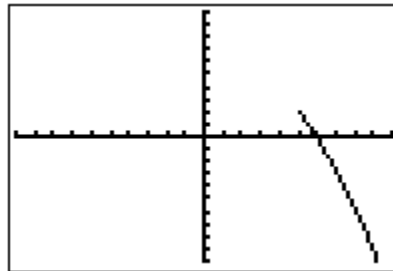
The greater than symbol (>) given in **Y1** above is accessed with **2nd |** **MATH(Test) | 3: >** as shown here.

Consider restricting the same function to the domain -7 < x < 5 by appending **/(x > -7 and x < 5)** to the end of the function specified by **Y1**.

```
       Plot2 Plot3
\Y1◼(-.2X²+7)/(X
>-7 and X<5)
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```

```
TEST LOGIC
1◼and
2:or
3:xor
4:not(
```
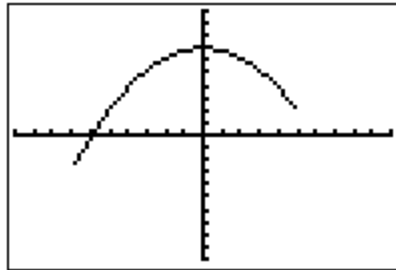
The logic specifier **and** given in **Y1** above is accessed with **2ⁿᵈ |** **MATH(Test) | Logic | 1: and** as shown here.

Consider restricting the same function to the domain x < -7 or x > 5 by appending **/(x < -7 or x > 5)** to the end of the function specified by **Y1**.

```
       Plot2 Plot3
\Y1◼(-.2X²+7)/(X
<-7 or X>5)
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```
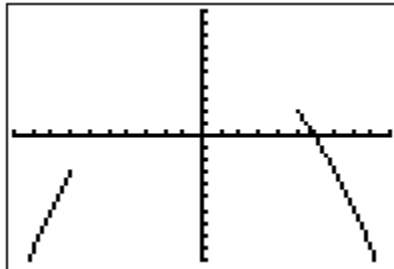
The logic specifier **or** given in **Y1** above is accessed with **2ⁿᵈ |** **MATH(Test) | Logic | 2: or**.

Calculator Appendix G

(Even and odd functions)

Consider the test to determine if a function is even: **f(x) = f(-x)**

This means if we graph f(x) it should be identical to f(-x).

A classical example of an even function is a parabola, for example, $f(x) = x^2 + 3$. If we graph $f(-x) = (-x)^2 + 3$, it should fall directly on top of f(x). For confirmation of this, enter both functions with **Y=** on a graphing calculator. The results are shown below:
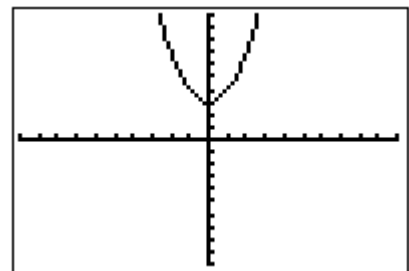
Consider the test to determine if a function is even: **f(x) = -f(-x)**

This means if we graph f(x) it should be identical to -f(-x).

A classical example of an even function is a cubic function, for example, $f(x) = .2x^3$. If we graph $f(-x) = -(.2(-x)^3)$, it should fall directly on top of f(x). For confirmation of this, enter both functions with **Y=** on a graphing calculator. The results are shown below:

Calculator Appendix H
(Inverse functions)

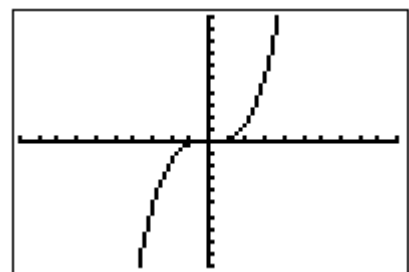The functions f(x) = 3x + 4 and g(x) = (x − 4)/3 are **inverse functions** of each other. This is confirmed when the two functions are simultaneously graphed. They are reflections of each other across a 45° line (y = x). All three are graphed below after entering the functions with **Y=**:

```
    ■■■■ Plot2 Plot3        WINDOW
\Y1■3X+4                     Xmin=-10
\Y2■(X-4)/3                  Xmax=10
\Y3■X                        Xscl=1
\Y4=                         Ymin=-10
\Y5=                         Ymax=10
\Y6=                         Yscl=1
\Y7=                         Xres=1
```
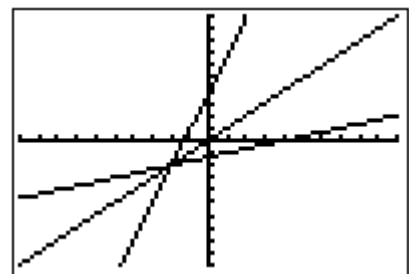
The above test for reflection across the line y = x is a **graphical test**. It would seem that on a graphing calculator that it would not be possible to do the **algebraic test** of f(g(x)) = x; however, it is possible if we enter

　　　Y1 = 3((x-4)/3) + 4   which is f(g(x))

If this graphs as a line through the origin with a slope of 1, then it is the line given by y = x. This is, in fact, the case as shown below, proving that they are inverses. (Actually, it would also be necessary to show that g(f(x)) also yields the line y = x.)

```
    ■■■■ Plot2 Plot3        WINDOW
\Y1■3((X-4)/3)+4            Xmin=-10
                            Xmax=10
\Y2=                        Xscl=1
\Y3=                        Ymin=-10
\Y4=                        Ymax=10
\Y5=                        Yscl=1
\Y6=                        Xres=1
```
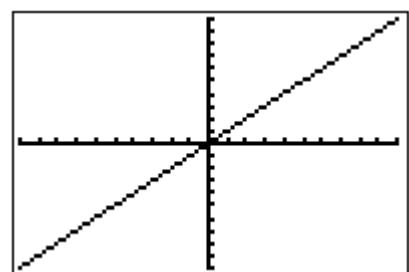
Calculator Appendix I

(Finding roots)

The root of a function is simply the x value of where it crosses the x-axis. It is possible for a function to have several roots.

Other names for **root** are **zero** and **x-intercept**. On a graphing calculator it is called a zero to emphasize that the value of the function is y = 0.

Consider the function $y = 3x^2 - 4x - 6$. Finding the roots of this function is equivalent to solving the equation $3x^2 - 4x - 6 = 0$.

Begin by pressing the **Y=** button and entering the function as **Y1**. Then press **GRAPH**.
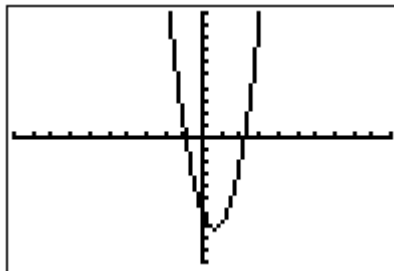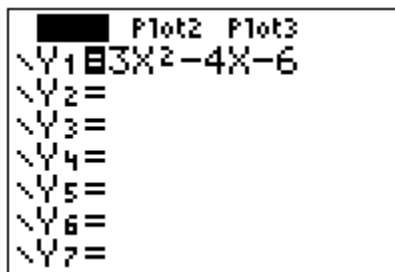
Clearly, from the graph there are two roots. We will concentrate on the left root.

To initiate the process of finding a root, press **2nd | CALC**. From the resulting menu choose **2: zero** by pressing the **DOWN ARROW** button thus moving the cursor down to **2:** and then pressing **ENTER**, or by just pressing the **2** button.
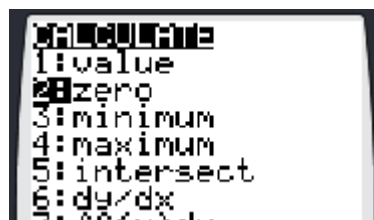
The next screen will show the graph along with a question, "Left Bound?". Move the blinker with the **LEFT/RIGHT ARROWS** until it is clearly to the left of the desired root (in this case, the left root). Press **ENTER** when satisfied with the position.
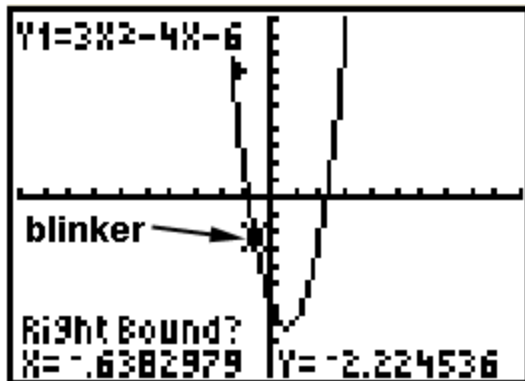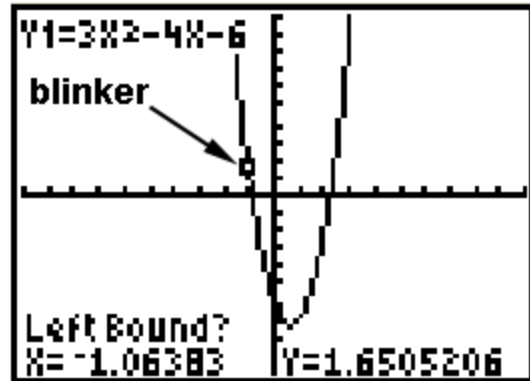
At this point another question will appear asking for the "Right Bound?". Move the blinker with the **LEFT/RIGHT ARROWS** until it is clearly to the right of the desired root. Press **ENTER**.

Finally, a third question is presented asking for a, "Guess?" Using the **LEFT/RIGHT ARROWS**, position the blinker approximately over the root and press **ENTER**.
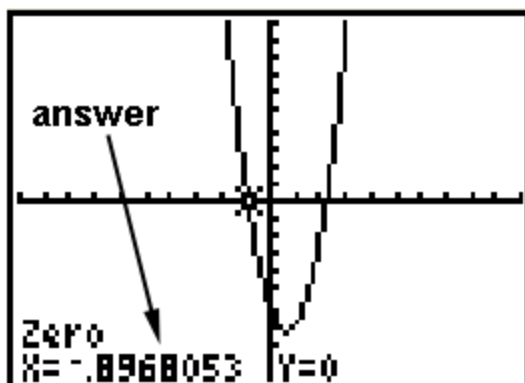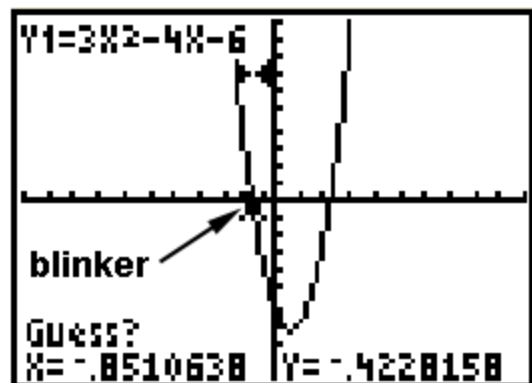
At this point the answer is presented at the bottom of the screen.

The other root is found in a similar fashion.

Calculator Appendix J

(Finding maximum and minimum points)

A **maximum point** of a function is a local point which has a greater value than all other points in its neighborhood. A function can have several maximums. A minimum point is similarly defined.

Consider the function $y = -x^2 - 12x - 30$. Our task is to find its single maximum. Obviously it's an "upside down" parabola and has a maximum point at the vertex of the parabola. Begin by pressing the **Y=** button and entering the function as **Y1**. Then press **GRAPH**.
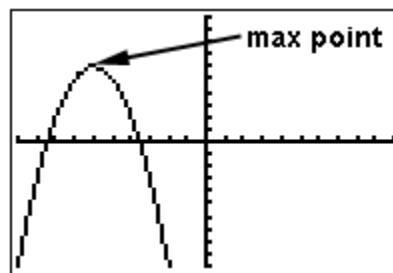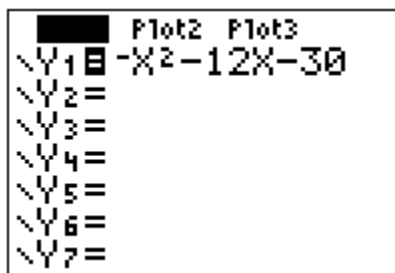
To initiate the process of finding the maximum point, press **2nd | CALC**. From the resulting menu choose **4: maximum** by pressing the **DOWN ARROW** button thus moving the cursor down to **4:** and then pressing **ENTER**, or by just pressing the **4** button.
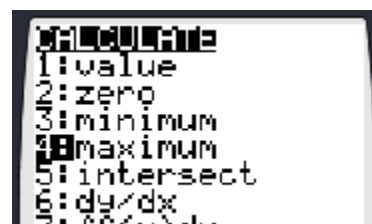
The instructions below will require moving the blinker with the **LEFT/RIGHT ARROWS**. Sometimes the blinker is not visible and it's not clear which way to move. Use the coordinates of the blinker at the bottom of the graph screen along with the zoom box coordinates to decide.
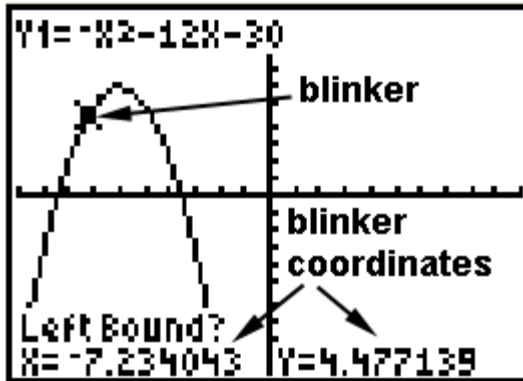
Y1=-X2-12X-30

blinker

blinker coordinates

Left Bound?
X=-7.234043   Y=4.477139

The next screen will show the graph along with a question, "Left Bound?". Move the blinker with the **LEFT/RIGHT ARROWS** until it is clearly to the left of the desired maximum point Press **ENTER** when satisfied with the position.

At this point another question will appear asking for the "Right Bound?". Move the blinker with the **LEFT/RIGHT ARROWS** until it is clearly to the right of the desired maximum point. Press **ENTER**.
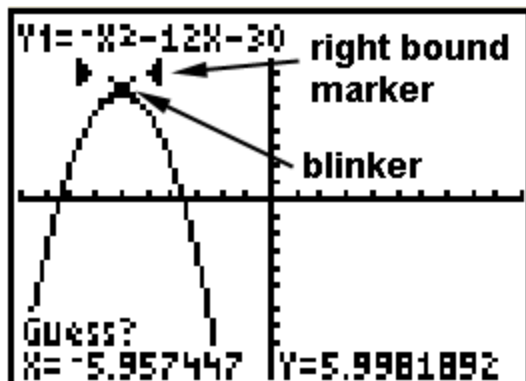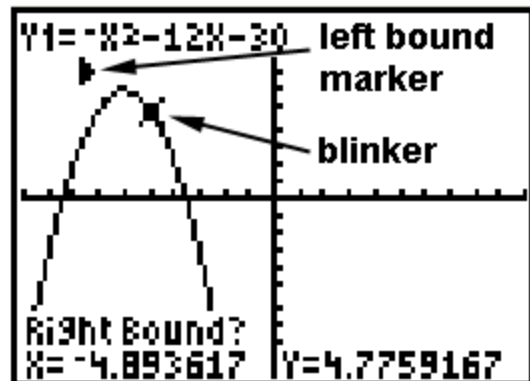
Y1=-X2-12X-30   left bound marker

blinker

Right Bound?
X=-4.893617   Y=4.7759167

Y1=-X2-12X-30   right bound marker

blinker

Guess?
X=-5.957447   Y=5.9981892

Finally, a third question is presented asking for a, "Guess?" Using the **LEFT/RIGHT ARROWS**, position the blinker approximately over the maximum point and press **ENTER**.

The coordinates of the maximum point are now presented at the bottom of the screen.

Minimum points are found in a similar fashion.

answer

Maximum
X=-5.999997   Y=6
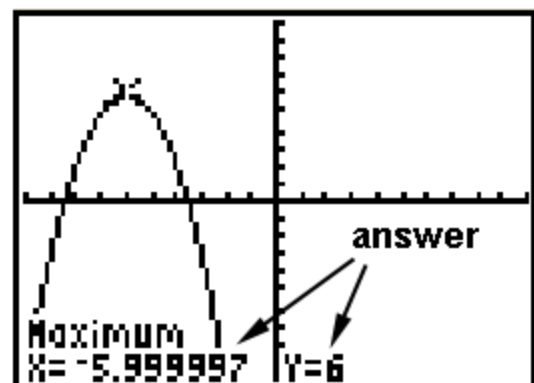
Calculator Appendix K

(Calculations with the natural number *e*)

Just as π is a natural number, so is *e*. The value of *e*, an irrational number, is approximately 2.718…

Typically, what we do with *e* is raise it to a power. Access this with **2ⁿᵈ |LN**. This places **e^(** on the screen which means *e* raised to the power in the parenthesis. Enter the argument, close the parenthesis, and press **ENTER** for the result.

To see *e* to 9 decimals places, enter simply 1 as the argument. This gives e^1 which, of course, is just *e*. Also shown here is an evaluation of $e^{4.6603}$.

```
e^(1)
         2.718281828
e^(4.6603)
         105.6677777
■
```

The reason *e* is associated with the LN button is that, that button gives the logarithm, base *e*. See Appendix L for a discussion of calculations of logarithms.

Press **Y=** and enter ln(x) for **Y1**. The graph of y = ln(x) is shown below. Notice that the curve **asymptotically approaches the y-axis** and has a **root of 1**.

```
        Plot2  Plot3
\Y1=ln(X)
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```
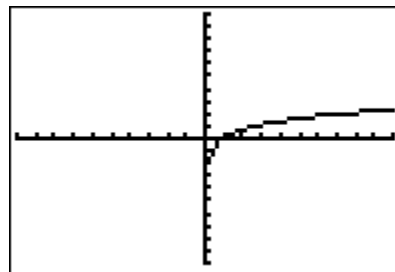
Press **Y=** and enter e^x for **Y1**. The graph of $y = e^x$ is shown below. Notice that the curve **asymptotically approaches the x-axis** and has a **y-intercept of 1**.
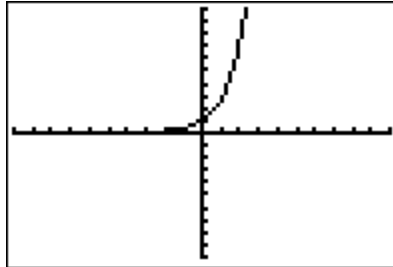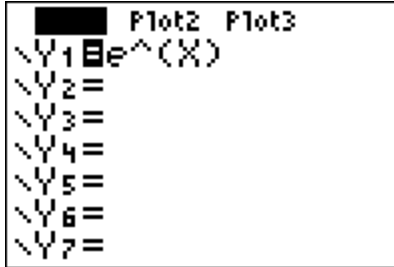
Calculator Appendix L

(Logarithms)

Two different choices are available when computing logarithms; base 10, and base *e*:

- Base 10 (common logarithms, use the **LOG** button)
- Base *e* (natural logarithms, use the **LN** button)

Calculate the common log of 100. This can be written as $\log_{10}100$ but is normally written as log 100. (When the base is omitted it's understood to be 10.)

The answer is 2 since this is the power to which 10 should be raised in order to equal 100. Shown here is how this problem is entered into the calculator. Notice that after pressing the **LOG** button, **log(** is displayed on the screen. Finish by entering 100 and a closing parenthesis. Press **ENTER** to get the answer (It's not necessary to close the parenthesis before pressing **ENTER,** but is highly recommended.)

log(100)

2

Similarly, calculate the natural log of 128.307. This can be written as $\log_e 128.307$ but is normally written as ln 128.307 or ln(128.307).

This problem is asking the question, "To what power must *e* be raised in order to get 128.307. (*e* is the irrational, natural number 2.718…)

Press the **LN** button which results in the display **ln(** . Finish entering the argument, 128.307, and press **ENTER**. The displayed results are shown here.

ln(128.307)
4.85442583

There are occasions in which a logarithm to a different base must be found. Consider the problem $log_{13}(153.6692)$. Use the change-of-base formula as follows:

$$log_{13}(153.6692) = \frac{\log{(153.6692)}}{\log{(13)}} \text{ , or}$$

$$log_{13}(153.6692) = \frac{\ln(153.6692)}{\ln(13)}$$

According to the formulas above, a logarithm with **any base** can be expressed in **terms of** either common or natural logs.

For common logs the display on the calculator would be as follows:

log(153.6692)/lo
g(13)
1.962924619

Calculator Appendix M

(Scatter Plots)

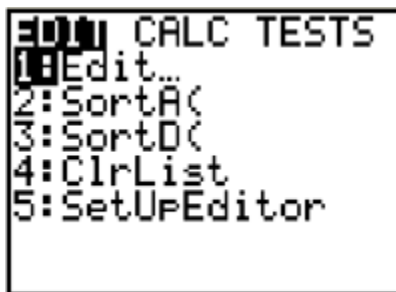| x	y
-9	-2.1
-5.2	-1
-2	2.04
2.3	5
6	6.11
8.9	8.2

Consider the data in this table. These points can be displayed as a "scatter-plot" with the steps described below.

The first step is to press the STAT button which yields the following display.

At this point either press **ENTER** or **1** indicating we wish to choose menu item 1 (editing/ entering the list of x and y values).

Enter the *x* and *y* values choosing each position with the **Left/Right** & **UP/DOWN ARROWS.** Enter the *x* values under list **L1** and the *y* values under list **L2**.

If **L1** and **L2** are not visible use the **LEFT/RIGHT ARROWS** so they display. If old lists are present us the **UP ARROW** to position the cursor on top of, for example **L1**, press **CLEAR**, and press **ENTER**.

The next step will be to press **2nd | STAT PLOT | 1 | On** so as to enable a scatter-plot to be displayed. This sequence is shown below. In the picture on the far right, use the **LEFT/RIGHT ARROWS** to choose between **On** and **Off** (choose **On**).

Finally, display the scatter plot by pressing the **GRAPH** button.

See Appendix N for how to produce the equation of a line that "best fits" this scatter-plot.

Calculator Appendix N

(Regression)

The process of regression is finding a function that "best-fits" a scatter-plot. In this Appendix a **linear regression** that determines the best-fit of a line will be demonstrated. Shown below is the data and the resulting scatter-plot from Appendix M that will be used here. That Appendix shows how to enter the data and produce the scatter-plot.
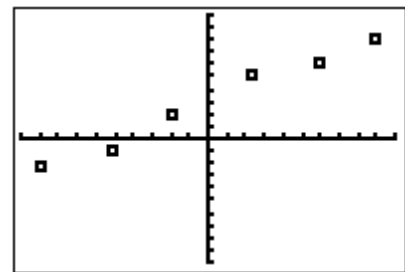
L1	L2	L3	2
-9	-2.1	------	
-5.2	-1		
-2	2.04		
2.3	5		
6	6.11		
8.9	8.2		
------	------	------	

L2(1)= -2.1

```
WINDOW
 Xmin=-10
 Xmax=10
 Xscl=1
 Ymin=-10
 Ymax=10
 Yscl=1
 Xres=1
```

With the above data already in the lists **L1** (the *x* values) and **L2** (the *y* values), press the **STAT** button. With the **RIGHT ARROW** move to **CALC** and then with the **DOWN ARROW** down to **4: LineReg(ax+b)**.

Press **ENTER** twice to produce the *a* and *b* values defining the best-fit line:

```
EDIT CALC TESTS
1:1-Var Stats
2:2-Var Stats
3:Med-Med
4:LinReg(ax+b)
5:QuadReg
6:CubicReg
7↓QuartReg
```

```
LinReg
 y=ax+b
 a=.5943811682
 b=2.942603139
```

At this point we could press **Y=** and enter:

Y1 = .5943811682X+2.942603139

However, there is an easier way. Rather than having to manually enter the *a* and *b* values for **Y1**, it is possible to automatically have them entered.

With the cursor on **Y1=** (on the **Y=** screen), press the **VARS** button, choose **5: Statistics**, **RIGHT ARROW** across to **EQ**, and then down to **1: RegEQ**. At this point the regression equation for the best-fit is automatically entered for **Y1**.
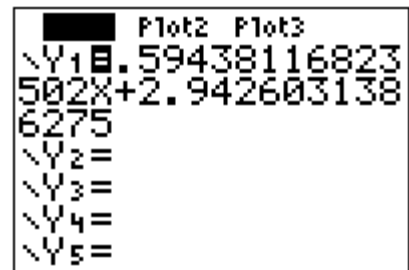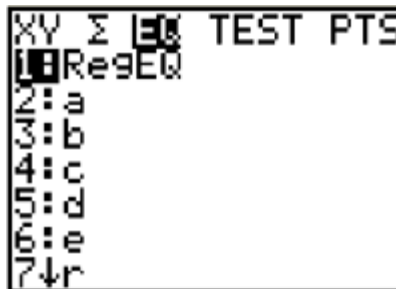
Press **GRAPH** and if **STAT PLOT** is still enabled (see Appendix M), the following should display as the best-fit for for the scatter-plot.
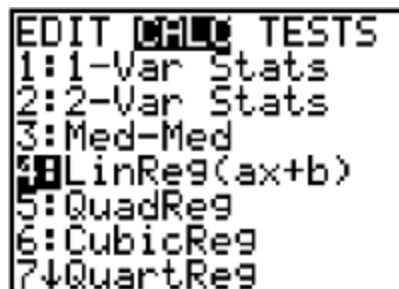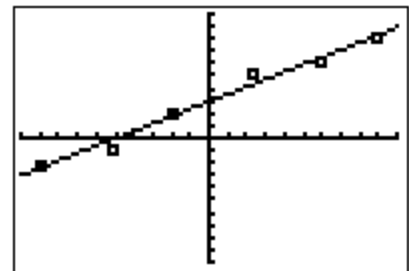
Some data does not lend itself to being best-fit with a line. For data that obviously "curves", other type regressions are available (parabolic (QuadReg), cubic, exponential, etc.)

When doing various regressions on a set of data points, it is possible to get "scores" on the quality of each fit.
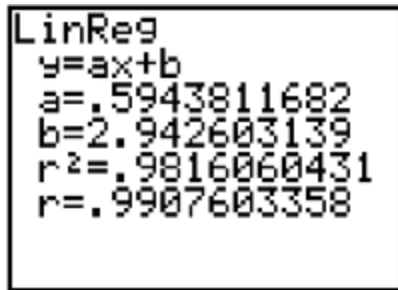
First, it will be necessary to turn on "diagnostics" so that a score can be displayed. Press **2ND| CATALOG**, **Down ARROW** down to **1: Diagnostics ON**, and press **ENTER** twice to turn on diagnostics.

With diagnostics on, a regression will now display r^2 = "some number less than or equal to one". Shown here is the same linear regression as above except now r^2 = .9816060431 is displayed along with a corresponding r value. r^2 is the square of the **correlation factor**, r, which can range from -1 to 1.

To eliminate the sometimes confusing negative r values, r^2 is often given as a figure of merit, were of, of course, r^2 = 1 is a **perfect score** as a result of a perfect fit.

For a linear regression, a negative r value indicates that the line of best-fit has a negative slope and the **correlation is said to be negative**. Likewise, a best-fit line with a positive slope is represented with a positive value of r and the **correlation is side to be positive**.
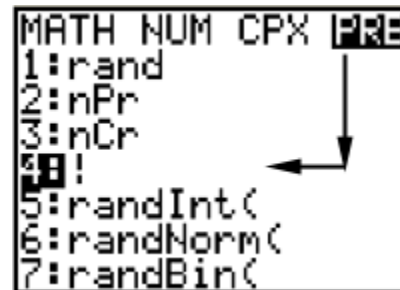
Calculator Appendix O
(Factorials, permutations, and combinations)

The factorial function is accessed via the **Math** button. For example, "four factorial" (written as 4!) evaluates as follows:

4! = 4(3)(2)(1) = 24.

Use the following sequence to evaluate, for example, 21! :

Enter 21, press the **Math** button, **RIGHT ARROW** across to **PRB**, **DOWN ARROW** to **4: !**, and press **ENTER**.
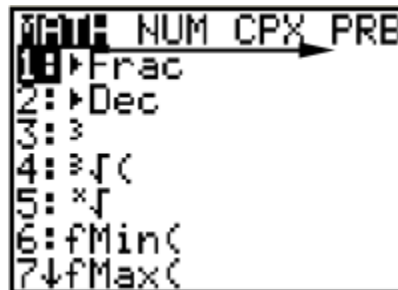
At this point you should have "21!" on the screen. Press **ENTER** one more time to evaluate this factorial:

21!
    5.109094217ᴇ19

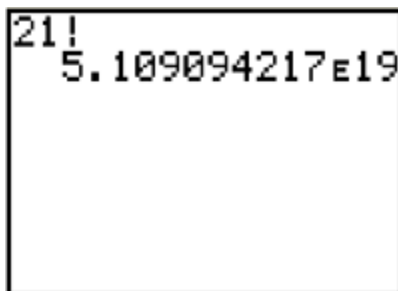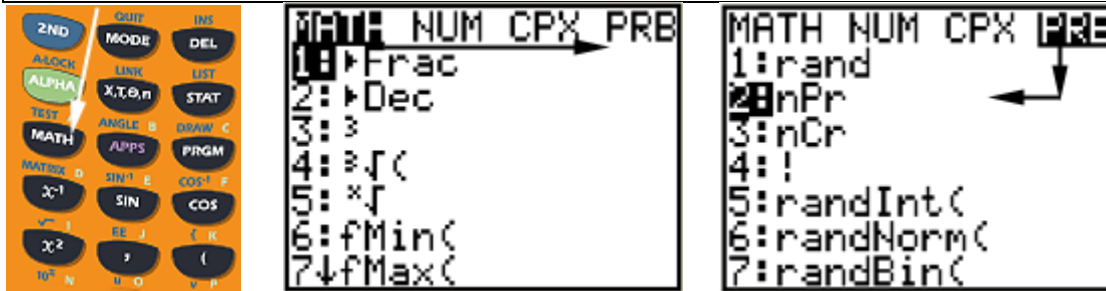The permutations function is also accessed via the **Math** button. For example, $_nP_r$ is read, "the number of permutations of n things taken r at a time". The formal definition is:

$$_nPr = \frac{n!}{(n-r)!}$$

Use the following sequence to evaluate, for example, $_{14}P_5$:

Enter 14, press the **Math** button, **RIGHT ARROW** across to **PRB**, **DOWN ARROW** to **2: nPr**, and press **ENTER**.

At this point you should have "14 nPr" on the screen. Enter 5 and the screen should now read "14 nPr 5". Press **ENTER** one more time to evaluate the number of permutations:


```
14 nPr 5
          240240
```

The combinations function is also accessed via the **Math** button. For example, nCr is read, "the number of combinations of n things taken r at a time". The formal definition is:
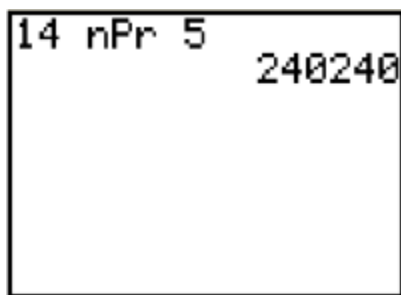
$$_n C_r = \frac{n!}{(n-r)! \, r!}$$

Use the following sequence to evaluate, for example, 56C45 :

Enter 14, press the **Math** button, **RIGHT ARROW** across to **PRB**, **DOWN ARROW** to **3: nCr**, and press **ENTER**.

At this point you should have "56 nCr" on the screen. Enter 45 and the screen should now read "56 nPr 45". Press **ENTER** one more time to evaluate the number of combinations:
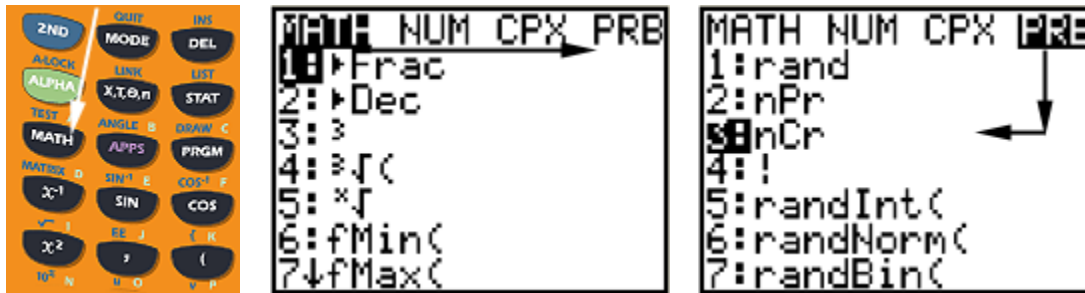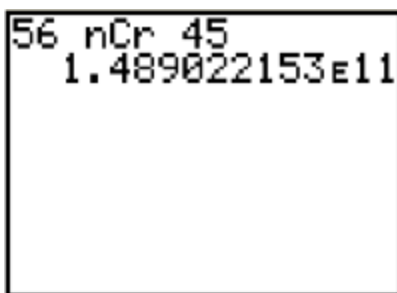
Calculator Appendix P

(Statistics)

Consider the following set of test grades:
{ 92, 99, 85, 95, 79, 64, 79, 91, 56, 82, 81 }

These numbers can be entered and the following statistics easily produced:

mean, median, standard deviation, lower-quartile, upper-quartile, min, max, … and more.

Press the **STAT** button and then select **EDIT** by pressing **ENTER**.

Under **L1**, enter the first number in the list, press the **DOWN ARROW**, and then enter the next number. Continue until all numbers are entered.(If **L1** is not initially empty, **UP ARROW** on top of **L1**, press **CLEAR** and then **ENTER**.)

Now that the data is entered we are ready to display statistical facts about the data:

Press the **STAT** button, **RIGHT ARROW** over to **CALC**.

Choose **1: 1-Var Stats**.
Press **ENTER** twice.

The statistics for the data in the **L1** list displays. Use the **DOWN-ARROW** to move down the display to view the remaining statistics.

```
1-Var Stats
 x̄=82.09090909
 Σx=903
 Σx²=75795      mean
 Sx=12.91088336
 σx=12.31004428
↓n=11
          standard dev
```

```
1-Var Stats
↑n=11
 minX=56
 Q₁=79 lower quartile
 Med=82 median
 Q₃=92 upper quartile
 maxX=99
```

The following definitions apply to the some of the displayed statistics:

The **mean** ($\bar{x}$) is the average:

The **median** is the number in the middle of the **ordered** list:
  (If there are two numbers in the middle, average them.)

The **lower quartile** is the median of the lower half of the numbers:

The **upper quartile** is the median of the upper half of the numbers:
  (When there are an odd number of data points, the middle value is not used when finding either quartile.)

The **standard deviation** (σ) is a measure of the average "scatter" of the data.

Calculator Appendix Q
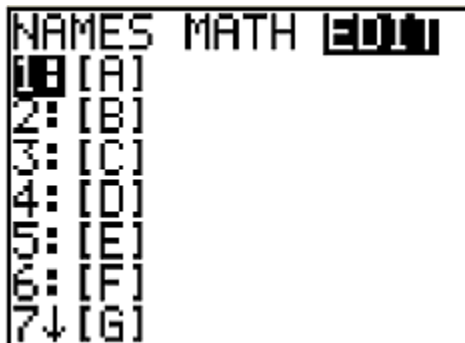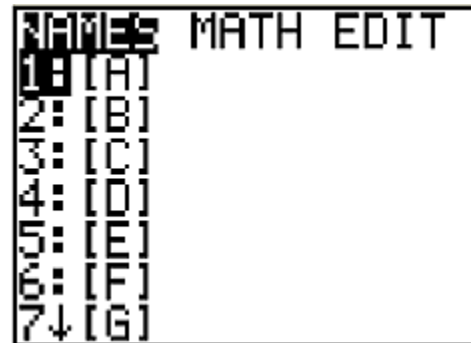
Matrix Fundamentals
(elementary matrix math)

Consider the following matrix problem:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \qquad C = \begin{bmatrix} -1 & -2 \\ -3 & -4 \end{bmatrix}$$

2A – 3(B – C)

The first step is to enter the matrices *A*, *B*, and *C*. Begin by pressing **2ND | MATRIX** resulting in this display. **RIGHT ARROW** twice across to **EDIT**.

Press **ENTER** and this screen appears. Use the **UP/DOWN ARROWS** to select the matrix we wish to edit (in this case **A**). Press **ENTER**.

This screen will appear asking for the dimensions of the **A** matrix in the upper right corner. Enter the number of rows first. Then press **ENTER**. Enter the number of columns and then **ENTER**.

```
MATRIX[A] 2 x2
[ 1        2    ]
[ 0        0    ]



2,1=3█
```
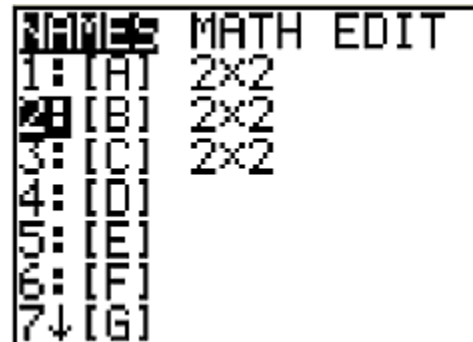
Enter each element of the **A** matrix pressing **ENTER** after each is entered. This display shows the next to last element (**3**) being entered.

Similarly, enter matrix **B** with the key sequence **2ND | MATRIX | EDIT | 2:[B] | ENTER.** In the same fashion, enter matrix **C**.

We are ready now to evaluate the matrix problem 2A – 3(B – C). Begin by entering 2. Place an **A** beside it using **2ND | MATRIX | Names | 1: [A] ENTER**.
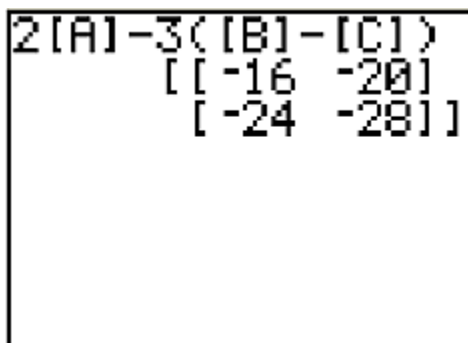
At this point we have **2A** on the screen. Next, enter **-3(** and the display should read **2A – 3(** .

B is entered with **2ND | MATRIX | Names | 2: [B] | ENTER** as shown here.

```
NAMES  MATH EDIT
1: [A]    2x2
2: [B]    2x2
3: [C]    2x2
4: [D]
5: [E]
6: [F]
7↓[G]
```

Finally, enter **– [C])** and press **ENTER**.

```
2[A]-3([B]-[C])
     [[-16  -20]
      [-24  -28]]
```

The final matrix answer is displayed.

Calculator Appendix R

Advanced Matrix Math:
## Multiplication, Inverse, Determinant

Matrices *A* and *B* can be **multiplied** in the order, *AB*, only if the number of columns in *A* is equal to the number of rows in *B*.

As an example, let $A = \begin{bmatrix} 1 & 3 & 2 & -1 \\ 2 & 1 & 0 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 4 & 1 \\ 3 & 1 & 2 \\ 1 & 1 & 3 \end{bmatrix}$
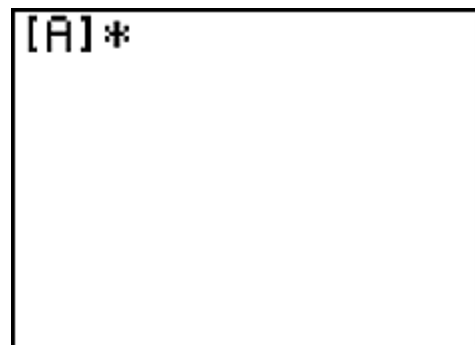
As described in **Calculator Appendix Q**, enter the *A* and *B* matrices.

Using **2nd MATRIX**, select the *A* matrix and then press **ENTER**.
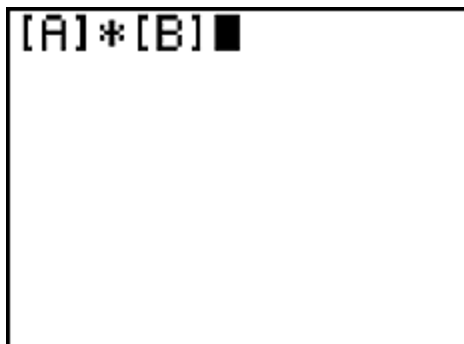
```
NAMES  MATH EDIT
1:[A]   2×4
2:[B]   4×3
3:[C]
4:[D]
5:[E]
6:[F]
7↓[G]
```

```
[A]■
```
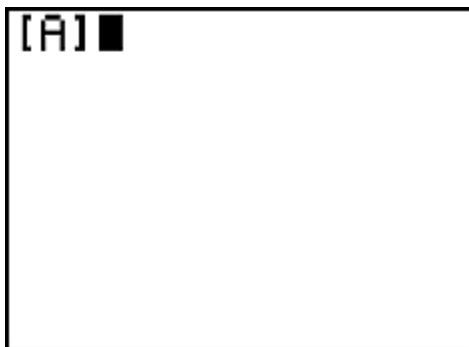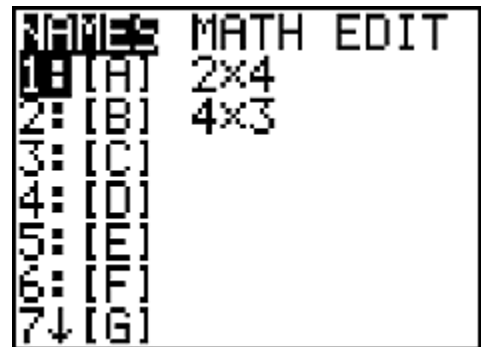
The resulting main screen display is shown here.

```
[A]*
```

We want to multiply this by the *B* matrix, so enter a times sign.

```
NAMES MATH EDIT
1:[A]  2×4
2:[B]  4×3
3:[C]
4:[D]
5:[E]
6:[F]
7↓[G]
```
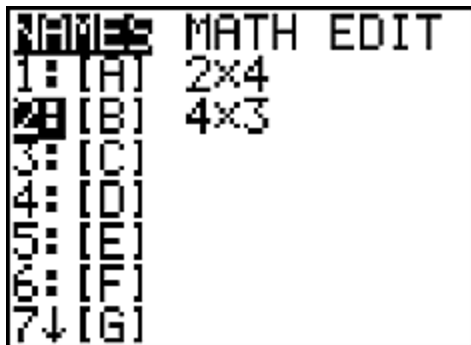
Using **2nd MATRIX | DOWN ARROW**, select the *B* matrix and press **ENTER**.

The main display will now look like this.

```
[A]*[B]■
```

```
[A]*[B]
    [[6 16 4 ]
     [0 8  -5]]
■
```
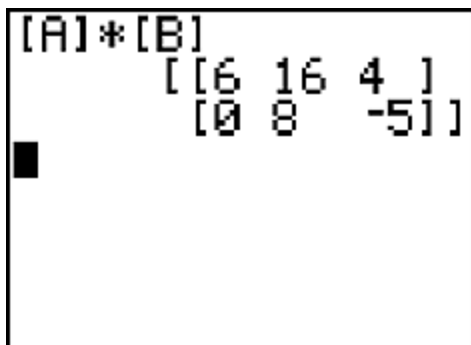
Press **ENTER** to obtain the final answer matrix.

****

To find the **inverse** of, for example, matrix $C = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$, first enter matrix *C* as described in **Calculator Appendix Q**.

Use **2nd MATRIX** and the **DOWN ARROW** to select matrix *C*.

```
NAMES MATH EDIT
1:[A]  2×4
2:[B]  4×3
3:[C]  2×2
4:[D]
5:[E]
6:[F]
7↓[G]
```

[C]
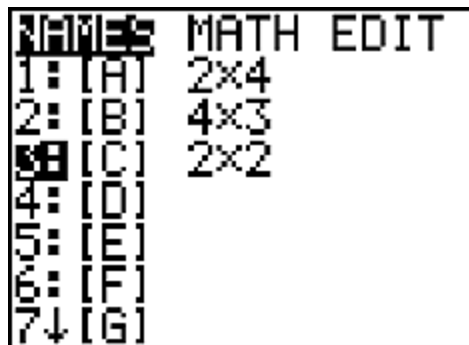
Press ENTER and obtain this display.

[C]⁻¹

[[⁻1 3]
 [1   ⁻2]]
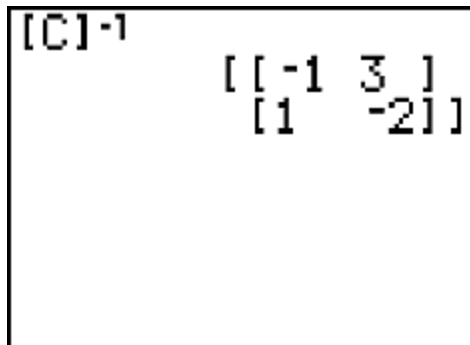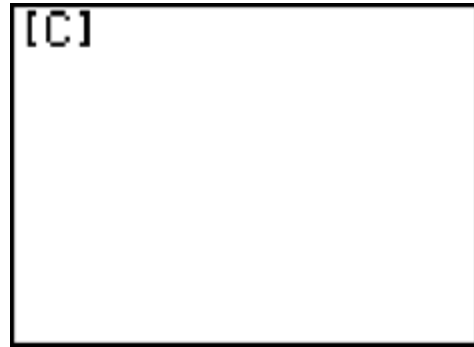
The inverse of *C* is represented symbolically by raising *C* to the -1 power.

Press the **X⁻¹** key and then **ENTER** to obtain the final answer, a matrix that is the inverse of *C*.

****

To find the **determinant** of, for example, matrix $C = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$, first enter matrix *C* as described in **Calculator Appendix Q**.

Use **2ⁿᵈ MATRIX | MATH | 1: det(| ENTER** to prepare to take the determinate of a matrix .

NAMES **MATH** EDIT
1█det(
2: ᵀ
3:dim(
4:Fill(
5:identity(
6:randM(
7↓augment(

```
det(
```

The main screen now displays this:

At this point we need to put the matrix C in the parenthesis.

```
NAMES MATH EDIT
1:[A]  2×4
2:[B]  4×3
3:[C]  2×2
4:[D]
5:[E]
6:[F]
7↓[G]
```

Using **2ⁿᵈ MATRIX | DOWN ARROW**, select the *C* matrix and then press **ENTER**.

Close the parenthesis and press **ENTER** to produce the answer.

```
det([C])
              -1
```