Blue Pelican Alg II

First Semester

Teacher Version 1.01

Alg II Syllabus (First Semester)

Unit 1: Solving linear equations and inequalities

Lesson 01: Solving linear equations

Lesson 02: Solving linear inequalities (See Calculator Appendix A and associated video.)

Lesson 03: *Solving combined (compound) inequalities

Lesson 04: Converting words to algebraic expressions

Lesson 05: Solving word problems with linear equations

Lesson 06: *Graphing calculator solutions of absolute value problems (See Calculator Appendix D & associated video, and Enrichment Topic A.)

Unit 1 review
Test: Unit 1 test

Unit 2: Slope; Solving a linear system of two equations

Lesson 01: Slopes of lines: four different points of view

Lesson 02: Two forms for the equation of a line

Lesson 03: Graphical meaning of the solution to two linear equations

Lesson 04: Algebraic solutions (elimination & substitution) for two linear equations

Lesson 05: Word problems involving two linear equations

Lesson 06: Graphing calculator solutions of linear systems (See Calculator Appendix C and associated video.)

Unit 2 review
Test: Unit 2 test

Unit 3: Graphing linear inequalities in two variables

Lesson 01: Graphing single linear inequalities in two variables

Lesson 02: Graphing systems of linear inequalities in two variables

Lesson 03: *Graphing calculator- graphing systems of linear inequalities in two variables (See Calculator Appendices B & E and associated videos. Also see Enrichment Topic B.)

Cumulative review, unit 3
Unit 3 review
Test: Unit 3 test

Unit 4: Multiplying and Factoring Polynomials

Lesson 01: Simple polynomial multiplication and factoring

Lesson 02: $(a + b)^2$, $(a – b)^2$, $(a – b)(a + b)$--- multiplying and factoring

Lesson 03: More trinomial factoring (Leading coefficient not one)

Lesson 04: Solving equations by factoring

Lesson 05: *Solving word problems with factoring

Lesson 06: *Binomial expansion theorem

Cumulative review, unit 4
Unit 4 review
Unit 4 test

Unit 5: Exponents and radicals

Lesson 01: Exponent rules (This lesson will likely span two days)

Lesson 02: Negative exponents

Lesson 03: More exponent problems

Lesson 04: Simplifying radical expressions

Lesson 05: Fractional exponents

Lesson 06: *Solving equations having rational & variable exponents

Lesson 07: *Solving radical equations

Lesson 8: Rationalizing denominators

Cumulative review, unit 5
Unit 5 review
Unit 5 test

Unit 6: Completing the square, the quadratic formula

Lesson 1: Solving equations by taking the square root

Topic M: Tangent composite function derivations

Topic N: Locating the vertex of a standard-form parabola

Topic O: Algebraic manipulation of inverse trig functions

Topic P: Logarithm theorem derivations

Topic Q: Arithmetic and geometric sum formulas

Topic R: Converting general form conics to standard form

Topic S: Conic section applications

Alg II, Unit 1

Solving linear equations and inequalities

Unit 1:
Lesson 01

Solving linear equations

Solve this linear equation: $3x - 4 = 5x + 1$

The goal is to first get all the x's on the left side of the equation and all the "other stuff" on the right side. In earlier courses we did this by first adding -5x to both sides as follows:

$$\begin{array}{rl} 3x - 4 &= \cancel{5x} + 1 \\ -5x \quad\;\; & \quad \cancel{-5x} \\ \hline -2x - 4 &= 1 \end{array}$$

In this course we will begin taking short-cuts to speed things along. Beginning again with 3x – 4 = 5x + 1, we say that we will "transpose" (move) the 5x to the left side. When we do, we change its sign. The result that is equivalent to the above is:

$$3x - 4 = 5x + 1$$
$$-5x + 3x - 4 = 1$$

Combine the –5x and 3x and then transpose the –4 to the right side and change its sign to get:

$$-2x - 4 = 1$$
$$-2x = 1 + 4$$
$$x = \boxed{\frac{5}{-2}}$$

Notice the last step where we divided both sides by –2.

In the following examples, solve for x:

Example 1: 2(4x – 9) = x +2

$$2(4x-9) = x+2$$
$$8x-18 = x+2$$
$$8x-x = 2+18$$
$$7x = 20$$
$$x = \boxed{\frac{20}{7}}$$

Example 2: x – 5x = 2(–3x + 4)

$$x-5x = 2(-3x+4)$$
$$-4x = -6x+8$$
$$-4x+6x = 8$$
$$2x = 8$$
$$x = \boxed{4}$$

Example 3: $\left(\frac{1}{2}\right)x + 6 = 14$

$$\left(\frac{1}{2}\right)x+6 = 14$$
$$\frac{1}{2}x = 14-6$$
$$\cancel{2}\frac{1}{\cancel{2}}x = 8\cdot 2$$
$$x = \boxed{16}$$

Example 4: $\frac{4(x-5)}{2}$ = 7x

$$\overset{2}{\cancel{4}}\frac{(x-5)}{\cancel{2}} = 7x$$
$$2x-10 = 7x$$
$$2x-7x = 10$$
$$-5x = 10$$
$$x = \frac{10}{-5} = \boxed{-2}$$

Assignment:

In problems 1 – 16, solve for the variable:

1. $12x = 24$

$x = \frac{24}{12}$

$x = \boxed{2}$

2. $x + 3 = 8$

$x + 3 = 8$

$x = 8 - 3$

$x = \boxed{5}$

3. $\frac{x}{2} = 11$

$\cancel{2} \cdot \frac{x}{\cancel{2}} = 11 \cdot 2$

$x = \boxed{22}$

4. $-11x = -33$

$x = \frac{-33}{-11}$

$x = \boxed{3}$

5. $-4 + \left(\frac{1}{3}\right)x = 4$

$3\left(-4 + \frac{1}{3}x\right) = 4 \cdot 3$

$-12 + x = 12$

$x = 12 + 12$

$x = \boxed{24}$

6. $3x = -2x + 10$

$3x = -2x + 10$

$3x + 2x = 10$

$5x = 10$

$x = \boxed{2}$

7. $-x - 2 = 22x + 1$

$-x - 2 = 22x + 1$

$-x - 22x = 1 + 2$

$-23x = 3$

$x = \boxed{-\frac{3}{23}}$

8. $7(x - 2) = 4(x + 15)$

$7(x - 2) = 4(x + 15)$

$7x - 14 = 4x + 60$

$7x - 4x = 60 + 14$

$\frac{3x}{3} = \frac{74}{3}$

$x = \boxed{\frac{74}{3}}$

9. $5(x-6)+1=4(-1+x)$

$5(x-6)+1=4(-1+x)$
$5x-30+1=-4+4x$
$5x-29=-4+4x$
$5x-4x=-4+29$
$x=\boxed{25}$

10. $12x-(x-1)=11+4x$

$12x-x+1=11+4x$
$11x+1=11+4x$
$11x-4x=11-1$
$7x=10$
$x=\boxed{\frac{10}{7}}$

11. $x(1+5)=x(2-7)+2$

$x(6)=x(-5)+2$
$6x+5x=2$
$11x=2$
$x=\boxed{\frac{2}{11}}$

12. $-(x+1)=3(x-6)$

$-(x+1)=3(x-6)$
$-x-1=3x-18$
$-x-3x=-18+1$
$-4x=-17$
$x=\frac{-17}{-4}=\boxed{\frac{17}{4}}$

13. $-8(-x-1)+7=2$

$-8(-x-1)+7=2$
$8x+8=2-7$
$8x=-5-8$
$8x=-13$
$x=\boxed{\frac{-13}{8}}$

14. $6x+2=\frac{3x+2}{2}$

$2(6x+2)=\frac{3x+2}{\cancel{2}}\cdot\cancel{2}$
$12x+4=3x+2$
$12x-3x=2-4$
$9x=-2$
$x=\boxed{\frac{-2}{9}}$

15. $\frac{6}{7}x + 1 = -x + 2$

$\frac{6}{7}x + 1 = -x + 2$

$\frac{6}{7}x + x = -1 + 2$

$\frac{13}{7}x = 1$

$x = \boxed{7/13}$

16. (3x + 8)/4 = 1 – x/2

$4\left(\frac{3x+8}{4}\right) = \left(1 - \frac{x}{2}\right)\cdot 4$

$3x + 8 = 4 - 2x$

$3x + 2x = 4 - 8$

$5x = -4$

$x = \boxed{-\frac{4}{5}}$

17. Simplify $3m + 2m - 6$

$\boxed{5m - 6}$

18. Simplify $-2(y-6)+8$

$-2(y-6)+8$

$= -2y + 12 + 8$

$= \boxed{-2y + 20}$

19. Simplify (11 – 2)y

$= (11-2)y$

$= \boxed{9y}$

or $= (11-2)y$

$= 11y - 2y$

$= \boxed{9y}$

20. Simplify $x(4+7)$

$= x(4+7)$

$= x \cdot 11 = \boxed{11x}$

or $= x(4+7)$

$= 4x + 7x$

$= \boxed{11x}$

Unit 1: Lesson 02 **Solving linear inequalities**

To solve an inequality, treat it just like an equation with this one exception: when multiplying or dividing both sides by a negative number, **reverse the inequality**.

It is suggested that when solving, the variable be isolated on the left side.

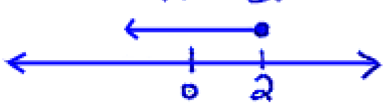
In the following examples, algebraically solve for x and then graph on a number line. (Be sure to graph $\le$ & $\ge$ with closed circles and $<$ & $>$ with open circles):

Example 1: $24\text{x} \le 48$

$24x \le 48$

$x \le \frac{48}{24}$

$x \le 2$



Example 2: $x - 8 < 5x$

$x - 8 < 5x$

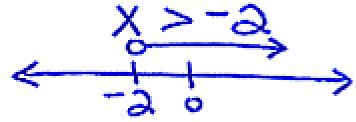
$-5x + x < 8$

$-4x < 8$

$x > \frac{8}{-4}$

$x > -2$

* Note sign reversal



Example 3: The teacher's age (*t*) is at least 3 times your age (*y*). Write this as an inequality in terms of *t* and *y*.

$t \ge 3y$

If instead of "at least," it said "at most," the answer would have been $t \le 3y$

Example 4: $6(x-4) \geq 22x-5$

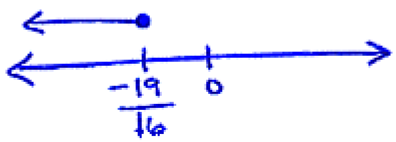
$6(x-4) \geq 22x-5$

$6x-24 \geq 22x-5$

$6x-22x \geq -5+24$

$-16x \geq 19$

(reversal) $x \leq \frac{19}{-16}$



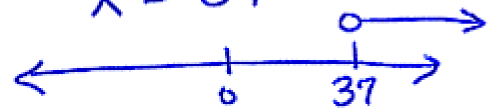
Example 5: $\frac{1}{2}(x-5) > 16$

$\cancel{2}\cdot\frac{1}{\cancel{2}}(x-5) > 16\cdot 2$

$x-5 > 32$

$x > 32+5$

$x > 37$

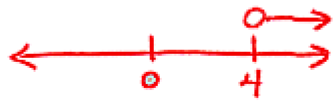


Assignment:

In problems 1 – 14, solve for the variable:

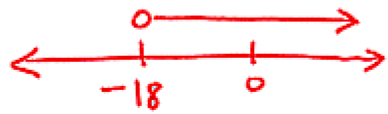
1. $x + 2 > 6$

$x + 2 > 6$
$x > 6 - 2$
$x > 4$



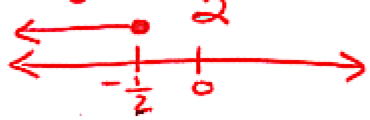
2. $\left(\frac{1}{3}\right) x > -6$

$\cancel{3} \cdot \frac{1}{\cancel{3}} x > -6 \cdot 3$
$x > -18$



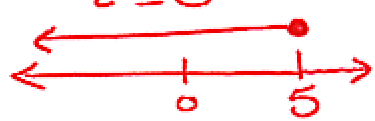
3. $2t - 3 \le -4$

$2t - 3 \le -4$
$2t \le -4 + 3$
$2t \le -1$
$t \le \frac{-1}{2}$



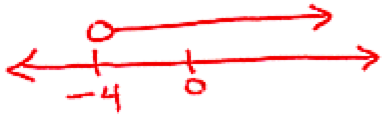
4. $-2z \ge -10$

$-2z \ge -10$
$z \le \frac{-10}{-2}$
$z \le 5$



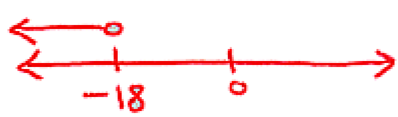
5. $7x + 4 > 5x - 4$

$7x + 4 > 5x - 4$
$7x - 5x > -4 - 4$
$2x > -8$
$x > -4$



6. $3x + 2 > 4(x + 5)$

$3x + 2 > 4(x + 5)$
$3x + 2 > 4x + 20$
$3x - 4x > 20 - 2$
$-x > 18$
$x < -18$ (reversal)

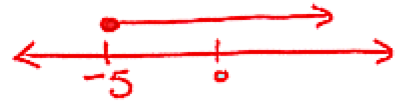


7. $2(p+2)-3>5(p-1)$

$2(p+2)-3>5(p-1)$
$2p+4-3>5p-5$
$2p+1>5p-5$
$2p-5p>-5-1$
$-3p>-6$
$p<2$ (reversal)

8. $4(1+m)\geq 2m-(1-m)$

$4(1+m)\geq 2m-(1-m)$
$4+4m\geq 2m-1+m$
$4+4m\geq 3m-1$
$4m-3m\geq -1-4$
$m\geq -5$

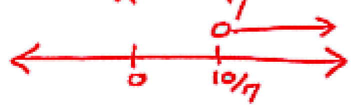


9. $3[2(x-1)-x]<5(x+2)$

$3[2(x-1)-x]<5(x+2)$
$3[2x-2-x]<5x+10$
$3[x-2]<5x+10$
$3x-6<5x+10$
$3x-5x<10+6$
$-2x<16$ (reversal)
$x>-8$

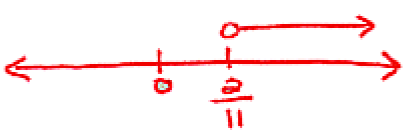
10. $12x-(x-1)>11+4x$

$12x-(x-1)>11+4x$
$12x-x+1>11+4x$
$11x+1>11+4x$
$11x-4x>11-1$
$7x>10$
$x>\frac{10}{7}$



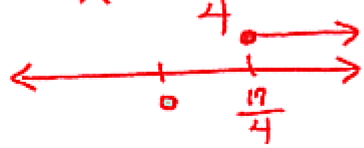
11. $x(1+5)>x(2-7)+2$

$x(6)>x(-5)+2$
$5x+6x>2$
$11x>2$
$x>\frac{2}{11}$



12. $-(x+1)\leq 3(x-6)$

$-(x+1)\leq 3(x-6)$
$-x-1\leq 3x-18$
$-x-3x\leq -18+1$
$-4x\leq -17$
$x\geq \frac{17}{4}$



13. My weight (m) is at least as much as your weight (y). Express this inequality in terms of m and y.

$m \geq y$

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14. Four times the number of hamburgers Bill ate (B) is still no more than what Lucas ate (L). Express this statement as an inequality in terms of B and L.

$4B \leq L$

Unit 1: Lesson 03

*Solving combined (compound) inequalities

In the following examples, algebraically solve for *x* and then graph on a number line.

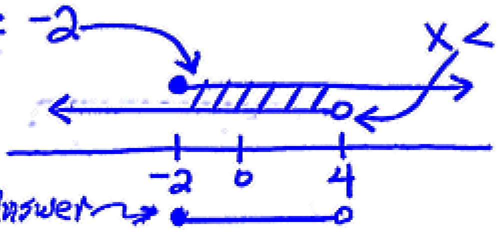
Example 1: $-4 \le 2\text{x} < 8$ First, we must recognize that this is really **two** simultaneous inequalities that could have been written as:

$-4 \le 2x$ *and* $2x < 8$ (also written as $-4 \le 2x$ ∩ 2x < 8)

Solve each half and combine:

$-2x \le 4$ $\quad\quad x < \frac{8}{2}$

$x \ge -2$ $\quad\quad x < 4$



Answer

Notice above that we wrote our problem as two separate inequalities with the word **and** (or the symbol ∩) between them. This is called a **conjunction** and makes it necessary to find the **intersection** of the two answers (where they overlap) to produce the final answer.

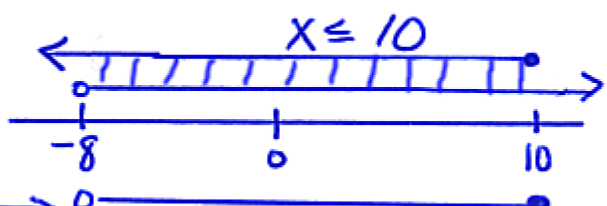
Example 2: $-4 < .5x \le 5$

$-4 < .5x$ *and* $.5x \le 5$

$-.5x < 4$ $\quad\quad x \le \frac{5}{.5}$

$x > \frac{4}{-.5}$ $\quad\quad x \le 10$

$x > -8$



Answer (Intersection)

In the next two examples the word "or" indicates a **disjunction** so we take the **union** of the two answers.

Example 3: $-4 > 2(x-1)$ or $x \geq 7$ (also written as $-4 > 2(x-1) \cup x \geq 7$)

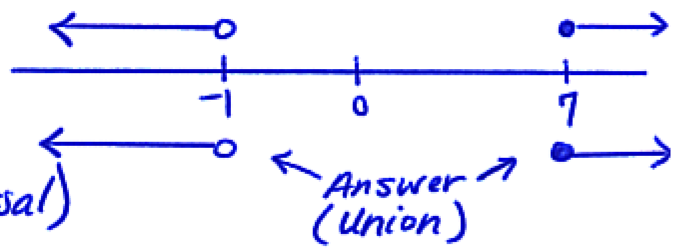
$-4 > 2x - 2$ or $x \geq 7$

$-2x > 4 - 2$

$-2x > 2$

$x < -1$

(reversal)



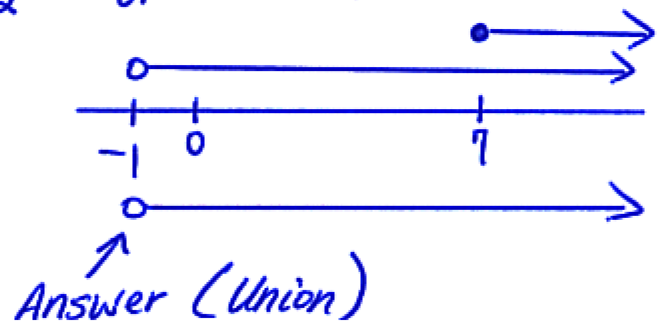
Example 4: $-4 < 2(x-1)$ or $x \geq 7$ $(-4 < 2(x-1) \cup x \geq 7)$

$-4 < 2x - 2$ or $x \geq 7$

$-2x < 4 - 2$

$-2x < 2$

$x > -1$



Assignment:

In problems 1 – 9, solve for the variable and graph on a number line:

1. $-2 \le x + 3 < 4$

$-2 \le x+3$ and $x + 3 < 4$

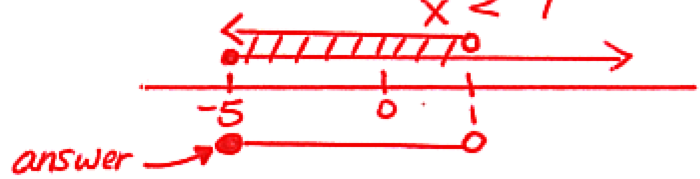
$-x \le 2+3$

$-x \le 5$

$x \ge -5$

$x < 4-3$

$x < 1$



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2. $z + 3 \le 1$ *or* $1 < z - 2$

$z+3 \le 1$ or $1 < z-2$

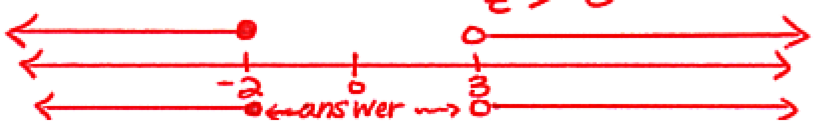
$z \le 1-3$

$z \le -2$

$-z < -1-2$

$-z < -3$

$z > 3$



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3. $1 \le m + 5 < 6$

$1 \le m+5$ and $m+5 < 6$

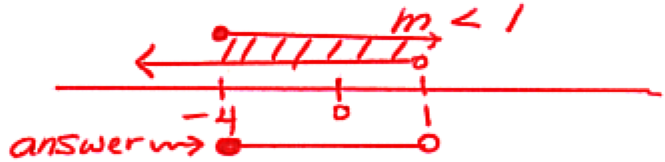
$-m \le 5-1$

$-m \le 4$

$m \ge -4$

$m < 6-5$

$m < 1$



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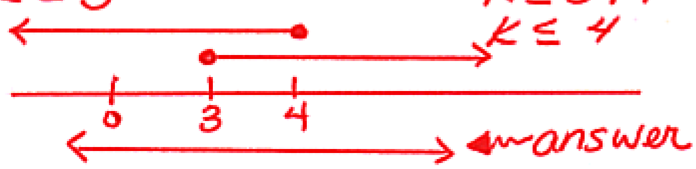
4. $k - 3 \ge 0 \;\; \cup \;\; k - 1 \le 3$

$k-3 \ge 0$ or $k-1 \le 3$

$k \ge 3$

$k \le 3+1$

$k \le 4$



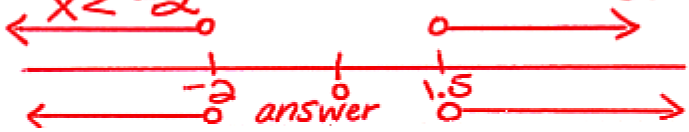
5. $1 - 2x > 5 \;\; or \;\; 1 - 2x < -2$

$1-2x>5$ or $1-2x<-2$

$-2x>5-1$ $\quad$ $-2x<-2-1$

$-2x>4$ $\quad$ $-2x<-3$

$x<-2$ $\quad$ $x>\frac{3}{2}$



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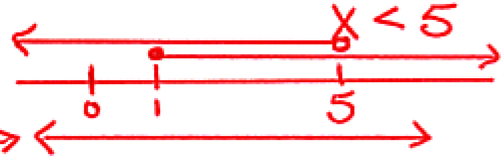
6. $3 - x \le 2 \;\; or \;\; x - 3 < 2$

$3-x\le 2$ or $x-3<2$

$-x\le 2-3$ $\quad$ $x<2+3$

$-x\le -1$ $\quad$ $x<5$

$x\ge 1$



answer ⇝

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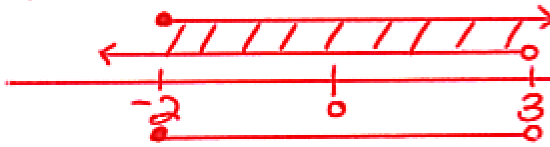
7. $2p - 2 < p + 1 \le 2p + 3$

$2p-2<p+1$ and $p+1\le 2p+3$

$2p-p<2+1$ $\quad$ $p-2p\le 3-1$

$p<3$ $\quad$ $-p\le 2$

$p\ge -2$



⇜ answer

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8. $6 > x - 1 > 3$

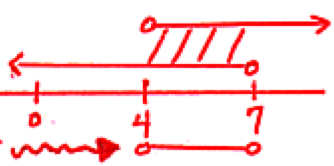
$6>x-1$ and $x-1>3$

$-x>-6-1$ $\quad$ $x>3+1$

$-x>-7$ $\quad$ $x>4$

$x<7$

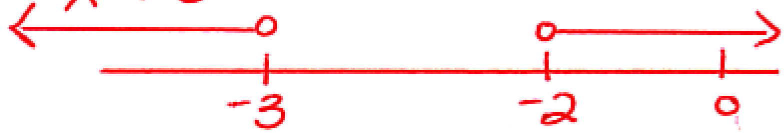
answer ⇝



9. $-3x > 9 \quad \cap \quad 8x > -16$

$-3x > 9$ and $8x > -16$

$x < -3$ $\qquad$ $x > -2$



$\varnothing$ No answer, they don't intersect

---

10. Answer these questions with either "and" or "or."

Conjunction is associated with what? and
Disjunction is associated with what? or

---

11. What symbol is used for intersection (and)? $\cap$
What symbol is used for union (or)? $\cup$

Unit 1: Lesson 04

Converting words to an algebraic expression

Before actually doing "word problems" where a variable is ultimately solved, it is necessary to become proficient at converting those words into mathematical language. **Define all variables.**

Example 1. What is the sum of three consecutive integers?

n = 1st int.
$n+1$ = 2nd int.
$n+2$ = 3rd int.

$n + n+1 + n+2$
$= \boxed{3n+3}$

Example 2. What is the product of three consecutive odd integers?

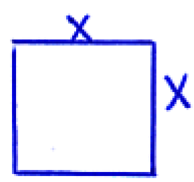
n = 1st odd int.
$n+2$ = 2nd " "
$n+4$ = 3rd " "

$\boxed{n(n+2)(n+4)}$

Example 3. Consider an isosceles triangle. What is the measure of the third angle if the two congruent angles measure *x* degrees?

$\boxed{180-2x}$

Example 4. What is the perimeter of a square with sides of length *x*?



$\boxed{4x = P}$

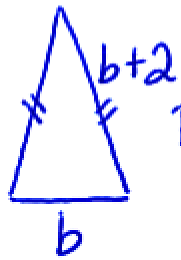
Example 5. A car travels for *h* hours at 50 mph. What is the total distance traveled?

dist = rate(time)
$= \boxed{50h}$

Example 6. What is the number that is 5 more than twice *n*?

$\boxed{2n+5}$

Example 7. What is the perimeter of an isosceles triangle whose base is *b* and the other two sides each being 2 cm longer than the base?



$$P = 2(b+2)+b$$
$$= 2b+4+b$$
$$= \boxed{3b+4}$$

Example 8. What is the cost of *t* movie tickets if the cost of each is $8.00?

$$\boxed{8t}$$

Assignment:

In the following problems, express the answer in terms of the variable given. Simplify when possible:

1. Let *n* be the first of 3 consecutive even integers. What is the sum of those integers?

$n = $ 1st even int.

$n+2 = $ 2nd " "

$n+4 = $ 3rd " "

$n + n+2 + n+4$

$= \boxed{3n+6}$

2. Let *i* be the first of four consecutive integers. What is 3 less than the sum of those integers?

$i = $ 1st int.

$i+1 = $ 2nd int.

$i+2 = $ 3rd int.

$i+3 = $ 4th int.

$i + i+1 + i+2 + i+3 - 3$

$= \boxed{4i+3}$

3. Consider four consecutive odd integers. What is the sum of the 2nd and fourth numbers if the first number is *n*?

$n = $ 1st odd int.

$n+2 = $ 2nd " "

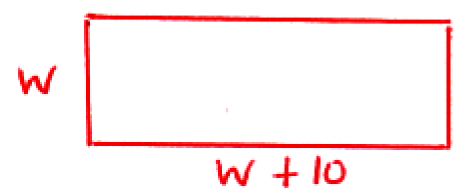
$n+4 = $ 3rd " "

$n+6 = $ 4th " "

$n+2+n+6$

$= \boxed{2n+8}$

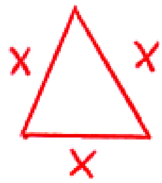
4. A playing field is 10 m longer that it is wide, *w*. What is its area?



$A = w(w+10)$

$= \boxed{w^2+10w}$

5. If the length of a side of an equilateral triangle is *x*, what is half of the perimeter?



$\boxed{\dfrac{3x}{2}}$

6. What is the distance traveled by a train moving at a rate of 45mph in *t* hours?

dist = rate (time)

$= \boxed{45t}$

7. Jim is twice as old as his sister. If *s* is the sister's age, what is the sum of their ages?

Jim's age = $2s$

Sister's age = s

$= 2s + s$

$= \boxed{3s}$

8. Three siblings are one year apart in age. If the oldest is *x* years old, what is the average of their ages?

x = oldest age

$x-1$ = middle age

$x-2$ = youngest age

Average = $\dfrac{\text{sum}}{3}$

$= \dfrac{x + x - 1 + x - 2}{3}$

$= \dfrac{3x-3}{3}$ or $\boxed{x-1}$

9. A car travels for 2 hours at *r* mph and then decreases its speed by 5 mph for the next 3 hours. What is the total distance traveled?

Total dist = $d_1 + d_2$

$= 2r + 3(r-5)$

$= 2r + 3r - 15$

$= \boxed{5r - 15}$

10. Two buses leave town at the same time going in opposite directions. Bus A travels at *v* mph while bus B is 10mph faster. How far apart are they in 3 hours?

Dist $= D_A + D_B$

$= 3v + 3(v+10)$

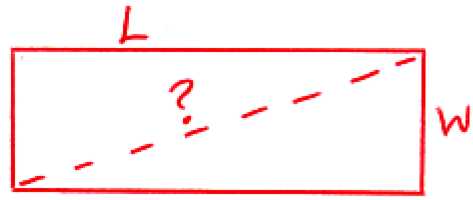
$= 3v + 3v + 30$

$= \boxed{6v + 30}$

11. A group of *g* students have detention and one girl left early. If 8 girls remain, how many boys remain?

$$boys = g - 1 - 8$$
$$= \boxed{g-9}$$

****12.** How long is the diagonal in a rectangle whose length, *L*, is three more than twice its width?



$$2w + 3 = L$$
$$2w = L - 3$$
$$w = (L-3)/2$$
$$Diag = \sqrt{L^2 + [(L-3)/2]^2}$$

13. What is the average of *y* plus 1 and three times *x*?

$$\boxed{\frac{3x + y + 1}{2}}$$

14. If a factory can produce *p* parts per hour, what would be the production in an 8 hour day?

$$production = rate(time)$$
$$= \boxed{8p}$$

Unit 1: Lesson 05 **Solving word problems with linear equations**

Read each problem, set up an equation with an appropriate variable and then solve.

Be sure to define the variable and make drawings when appropriate.

Example 1: Find three consecutive odd integers whose sum is 81.

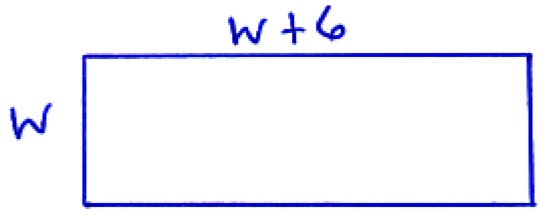
n = 1st odd integer = $\boxed{25}$
$n+2$ = 2nd " " = $\boxed{27}$
$n+4$ = 3rd " " = $\boxed{29}$
answers

$n+n+2+n+4=81$
$3n+6=81$
$3n=81-6$
$3n=75$
$n=25$

Example 2: An angle is 8 times its supplement. What is the angle?

A = the angle = $\boxed{160}$
$180-A$ = its supplement
$A=8(180-A)$
$A=1440-8A$
$A+8A=1440$
$9A=1440$
$A=\frac{1440}{9}=160$

Example 3: A board game is 6 cm longer than it is wide. If the perimeter is 152 find both dimensions.



$P = 2(w+6) + 2w = 152$

$2w + 12 + 2w = 152$

$4w = 152 - 12$

$4w = 140$

$w = 140/4 = 35$

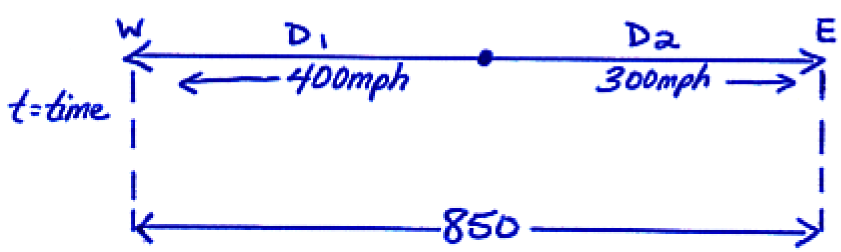
$w = \boxed{35}$

$L = w + 6$

$= 35 + 6$

$= \boxed{41}$

Example 4: Two planes leave an airport at 1:00 PM and fly in opposite directions. The plane going East flies at 300 mph and the one going West has a speed of 400 mph. How long does it take for them to be 850 miles apart? *According to the clock, what time will that be?



$D_1 + D_2 = 850$

Dist = rate(time)

$400t + 300t = 850$

$700t = 850$

$t = 1.21428$

$t = \boxed{1.21 \text{ hrs.}}$

$.21(60) = 12.6$ min.

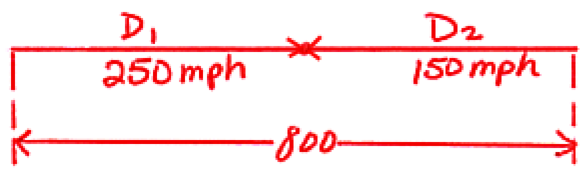
Clock time $= 1{:}00 + 12.6 \approx \boxed{2{:}13 \text{ PM}}$

Assignment:

Read each problem, set up an equation with an appropriate variable and then solve. Be sure to define the variable and make drawings when appropriate.

1. Two planes that are initially 800 mi apart are flying directly toward each other. One flies at 250 mph and the other at 150 mph. How long will it be before they meet?

t = time



$$D_1 + D_2 = 800$$
$$250t + 150t = 800$$
$$400t = 800$$
$$t = \boxed{2\text{ hrs.}}$$

---

2. Bill's number is 4 more than Mary's. The average of their numbers is 36. What is each person's number?

Bill's # $= n+4 = \boxed{38}$
Mary's # $= n = \boxed{34}$

$$\frac{n+4+n}{2} = 36$$
$$\cancel{2}\cdot\frac{2n+4}{\cancel{2}} = 36\cdot 2$$
$$2n+4 = 72$$
$$2n = 72-4$$
$$2n = 68$$
$$n = 34$$

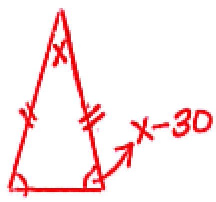
---

3. Initially the first bank account is \$5 less than the second account. After a month the first account had tripled and the second account had doubled. At that time they were equal. What was the amount originally in each account?

First account $= x-5 = 15-5 = \boxed{10}$
Second account $= x = \boxed{15}$

$$3(x-5) = 2x$$
$$3x-15 = 2x$$
$$3x-2x = 15$$
$$x = 15$$

4. In an isosceles triangle the base angles are 30 degrees less than the other angle. What is the measure of the third angle?



$$x + 2(x-30) = 180$$
$$x + 2x - 60 = 180$$
$$3x = 180 + 60$$
$$3x = 240$$
$$x = \boxed{80}$$

---

5. The sum of 3 consecutive integers is 153. What are the three integers?

n = 1st integer = 50
$n+1$ = 2nd " = 51
$n+2$ = 3rd " = 52

answers

$$n + n+1 + n+2 = 153$$
$$3n + 3 = 153$$
$$3n = 153 - 3$$
$$3n = 150$$
$$n = 50$$

---

6. The sum of 4 consecutive even integers is 44. What is the largest of those four integers?

n = 1st even int.
$n+2$ = 2nd " "
$n+4$ = 3rd " "
$n+6$ = 4th " "
$= 8 + 6 = \boxed{14}$

$$n + n+2 + n+4 + n+6 = 44$$
$$4n + 12 = 44$$
$$4n = 44 - 12$$
$$4n = 32$$
$$n = 8$$

---

7. An angle is exactly half its complement. What is the angle?

A = the angle
$90 - A$ = its complement

$$A = \tfrac{1}{2}(90 - A)$$
$$A = 45 - \tfrac{1}{2}A$$
$$2A = 2(45 - \tfrac{1}{2}A)$$
$$2A = 90 - A$$
$$2A + A = 90$$
$$3A = 90$$
$$A = \boxed{30}$$

8. A number is multiplied by 2 and then this product is increased by 5. The result is 14 less than twice the opposite of the number. What is the original number?

n = the number

$$2n+5 = 2(-n)-14$$
$$2n+5 = -2n-14$$
$$2n+2n = -14-5$$
$$4n = -19$$
$$\frac{4n}{4} = \frac{-19}{4}$$
$$n = \boxed{\frac{-19}{4}} \text{ or } \boxed{-4.75}$$

---

9. The average of three consecutive integers is 17. What are the integers?

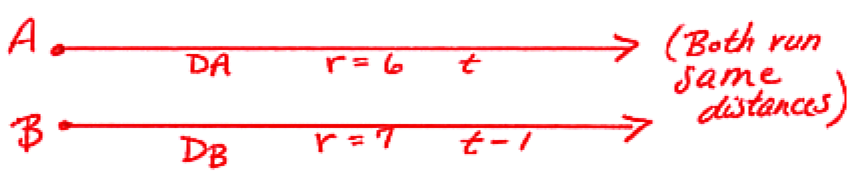
n = 1st integer = $\boxed{16}$
$n+1$ = 2nd " = $\boxed{17}$
$n+2$ = 3rd " = $\boxed{18}$

answers

$$\frac{n+n+1+n+2}{3} = 17$$
$$\cancel{3}\cdot\frac{3n+3}{\cancel{3}} = 17\cdot 3$$
$$3n+3 = 51$$
$$3n = 51-3 = 48$$
$$n = \boxed{16}$$

---

10. Long distance runner A averages 6 mph, while runner B averages 7 mph. They both start from the same starting position; however, runner B begins an hour after A leaves. How long will it take B to catch A?



$$DA = DB$$
$$6t = 7(t-1)$$
$$6t = 7t-7$$
$$6t-7t = -7$$
$$-t = -7$$

$t = \boxed{7 \text{ hours}}$

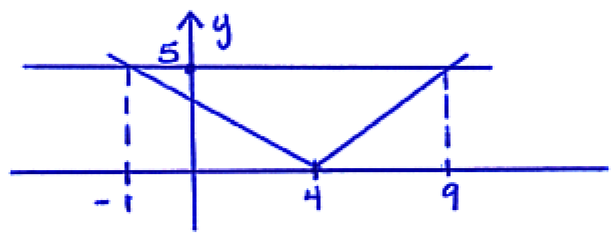
Unit 1: Lesson 06 **Graphing calculator solutions: absolute value problems**

Consider the solutions to the three problems: $|x-4|=5$, $|x-4|\leq 5$, and $|x-4|\geq 5$. To solve these on the calculator, all will require us to graph two functions:

Y1 = abs(X – 4) (absolute value bars are replaced with "abs" from the **Math | Num** menu)

Y2 = 5

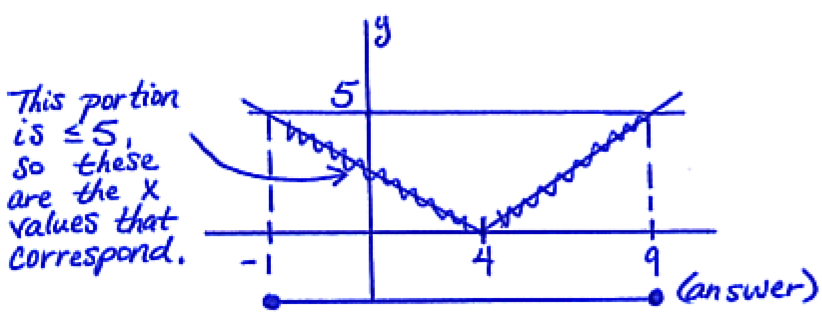
Using a "6. ZStandard" zoom, the simultaneous graph of these two is the following where the intersection points have been determined on the calculator with **2nd Calc | intersect** (See **Calculator Appendix D** and a related video for instructions.):



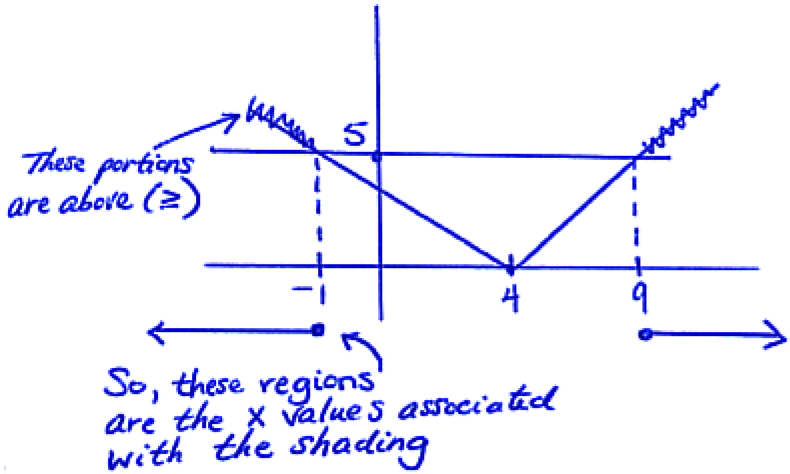
Example1: $|x-4|=5$

For this problem, the answers are –1 and 9 as determined in the drawing above.

Example 2: $|x-4|\leq 5$



Example 3: $|x - 4| \geq 5$



Now consider how to **manually** solve a linear equation involving the absolute value of a variable expression.

Example 4: $|x - 2| = 3$ When an absolute value is **equal** to a number, create two separate equations as follows and take the union (**or**) of the answers.

$x - 2 = 3$ or $x - 2 = -3$

$x = 3 + 2$ $\qquad$ $x = -3 + 2$

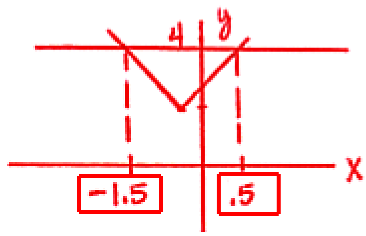
$x = \boxed{5}$ $\qquad$ $x = \boxed{-1}$

two answers

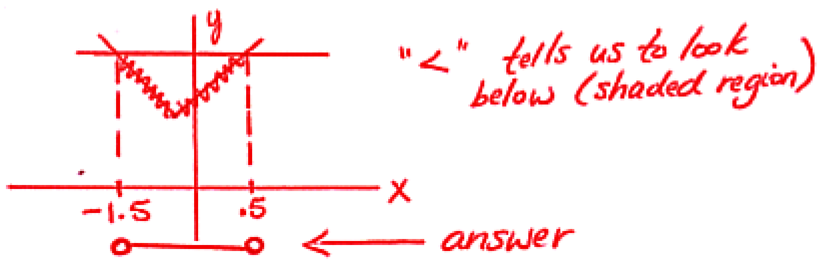
See **Enrichment Topic A** for how to **manually solve inequalities** involving the absolute value of variable expressions.

Assignment:
Solve problems 1-3 with a graphing calculator. Make a sketch of the x-y plot and finally draw the solution to the problem on a number line.

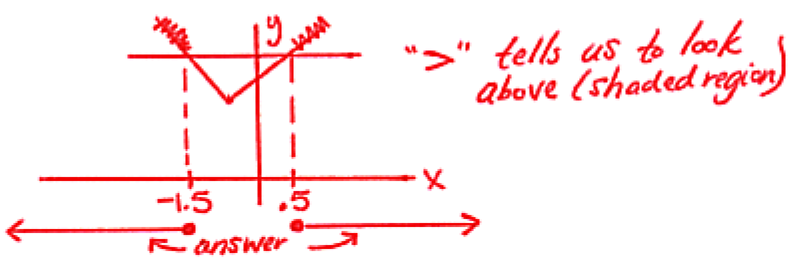
1. $|-2x-1|+2=4$



2.$|-2x-1|+2<4$



3. $|-2x-1|+2\geq 4$



Solve the following three problems **manually** (no calculator).

4. $|x+6|=17$

$x+6=17$ or $x+6=-17$

$x=17-6$ $\quad x=-17-6$

$x=\boxed{11}$ $\quad x=\boxed{-23}$

5. $|2t-3|=4$

$2t-3=4$ or $2t-3=-4$

$2t=4+3$ $\quad 2t=-4+3$

$2t=7$ $\quad 2t=-1$

$t=\boxed{\frac{7}{2}}$ $\quad t=\boxed{\frac{-1}{2}}$

6. $|-2z+8|=10$

$-2z+8=10$ $\quad -2z+8=-10$

$-2z=10-8$ $\quad -2z=-10-8$

$-2z=2$ $\quad -2z=-18$

$z=\boxed{-1}$ $\quad z=\boxed{9}$

Review for test on Unit 1

Unit 1: Review

In problems 1- 4, solve for the variable and show all work:

1. 4(x –7) = x + 2

$4x-28=x+2$
$4x-x=2+28$
$3x=30$
$x=\boxed{10}$

2. – 6(x – 5) = x + 5

$-6x+30=x+5$
$-6x-x=5-30$
$-7x=-25$
$x=\boxed{\frac{25}{7}}$

3. 2x(6 – 3) + 4 = x –2

$2x(3)+4=x-2$
$6x+4=x-2$
$6x-x=-2-4$
$5x=-6$
$x=\boxed{-\frac{6}{5}}$

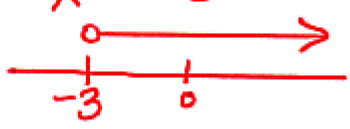
4. $\frac{3(2-x)}{2}=-8$

$\cancel{2}\cdot\frac{3(2-x)}{\cancel{2}}=-8\cdot 2$
$3(2-x)=-16$
$6-3x=-16$
$-3x=-16-6$
$-3x=-22$
$x=\boxed{\frac{22}{3}}$

In problems 5 and 6, algebraically solve for the variable and graph the answer on a number line. Show your work.

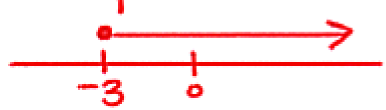
5. 5(x + 7) > 20

$5x+35>20$
$5x>20-35$
$5x>-15$
$x>-3$



6. –3(p + 1) ≤ –2p

$-3p-3\le -2p$
$-3p+2p\le 3$
$-p\le 3$
$p\ge -3$



7. Write an expression for Bob's age. He is 5 years older than 3 times Amy's age (A).

A = Amy's age

$B = \boxed{3A+5}$

8. Make two associations groups from the words intersection, union, disjunction, conjunction, and, & or.

Intersection	Union
Conjunction	Disjunction
And	Or

9. If x is the first even number, write an expression for the average of three consecutive even numbers.

x = 1st even intg.

x+2 = 2nd " "

x+4 = 3rd " "

$\text{Avg.} = \frac{x+x+2+x+4}{3}$

$\text{Avg.} = \boxed{\frac{3x+6}{3}}$ or x+2

simplified

(not necessary at this point)

10. An angle is one more than twice its complement. What is the angle?

A = the angle

90 − A = its complement

$A = 2(90-A)+1$

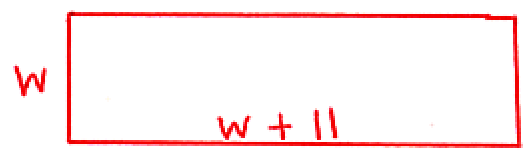
$A = 180-2A+1$

$A+2A = 181$

$3A = 181$

$A = \boxed{\frac{181}{3}}$

11. The new court house is 11 meters longer than it is wide. The perimeter of the court house is 175 meters. What are its dimensions?



$L = 38.25+11$

$= \boxed{49.25}$

$2(W+11)+2W = 175$

$2W+22+2W = 175$

$4W = 175-22$

$4W = 153$

$W = \boxed{\frac{153}{4} = 38.25}$

12. If twice the amount of flour in the pantry exceeds the amount of corn meal by two pounds, what is the amount of flour? There are 15 pounds of corn meal in the pantry.

F = amount of flour

$2F = 15+2$

$2F = 17$

$F = \boxed{\frac{17}{2}}$

**13. Solve |3x –1| = 11 for x.

$3x-1=11$ $\quad\quad$ $3x-1=-11$

$3x=11+1$ $\quad\quad$ $3x=-11+1$

$3x=12$ $\quad\quad$ $3x=-10$

$x=\boxed{4}$ $\quad\quad$ $x=\boxed{\frac{-10}{3}}$

*14. Solve $-3 \le x - 1 < 6$ for x. Solve algebraically and then present the answer both algebraically and on a number line.

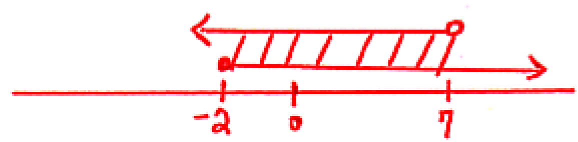
$-3 \le x-1$ $\quad$ and $\quad$ $x-1<6$

$-x \le 3-1$ $\quad\quad$ $x<6+1$

$-x \le 2$ $\quad\quad$ $x<7$

$x \ge -2$

$x \ge -2$ and $x<7$



*15. Solve $-3 \ge x - 1 \quad or \quad x + 3 > 7$ for x. Solve algebraically and then present the answer both algebraically and on a number line.

$-3 \ge x-1$ $\quad$ or $\quad$ $x+3>7$

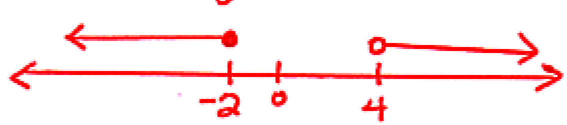
$-x \ge 3-1$ $\quad\quad$ $x>7-3$

$-x \ge 2$ $\quad\quad$ $x>4$

$x \le -2$

$\boxed{x \le -2 \text{ or } x>4}$

Alg. Solution



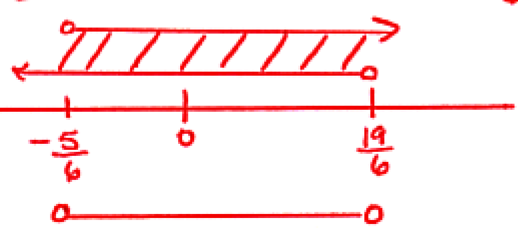
16. **This problem is from Enrichment Topic A: Solve $|7 - 6x| < 12$ for x. Solve algebraically and then present the answer both algebraically and on a number line.

$7 - 6x < 12$ and $7 - 6x > -12$

$-6x < 12 - 7$ $-6x > -12 - 7$

$-6x < 5$ $-6x > -19$

$x > \frac{-5}{6}$ $x < \frac{19}{6}$



$-\frac{5}{6} < x < \frac{19}{6}$

Alg II, Unit 2

Slope, solving a linear system of two equations

Unit 2: Lesson 01

Slopes of lines: four different points of view

The slope (m) of a line is simply a measure of its **steepness**; therefore, a **horizontal line has a slope of m = 0** since it has no steepness. A **vertical line** is the ultimate in steepness so its slope is **infinity** (undefined).

What about lines that are neither horizontal nor vertical? We are going to look at how to determine the slope four different ways.

First method: The two-point slope formula

Given two points (x_1, y_1) and (x_2, y_2) $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$

Example 1:

Find the slope of the line connecting the points (2, 3) and (5, –6).

$x_1 y_1 \quad x_2 y_2$

$m = \frac{-6-3}{5-2} = \frac{-9}{3} = \boxed{-3}$

Example 2:

Find the slope of the line connecting the points (–2, 4) and (–2, –17).

$x_1 \ y_1 \quad x_2 \ y_2$

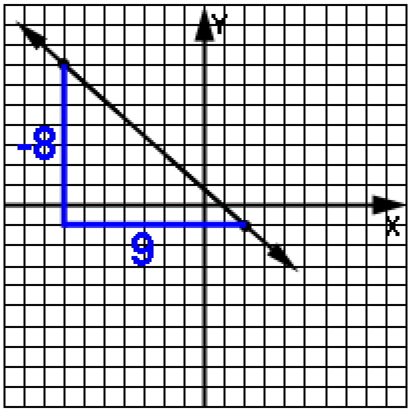
$m = \frac{-17-4}{-2+2} = \frac{-21}{0} =$ Forbidden!

Vertical line, $\boxed{\text{no slope}}$

Second method: $\quad m = \frac{\text{rise}}{\text{run}}$

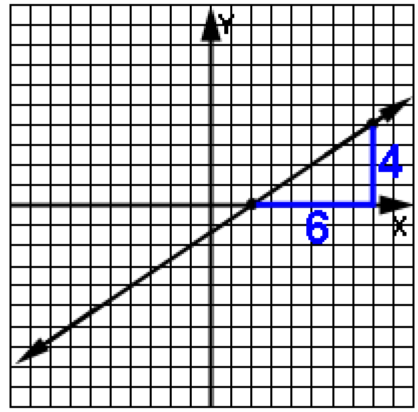
Simply find two places on the line and create a triangle. The horizontal line segment is the run (always considered positive) while the vertical segment is the rise. It can be either positive or negative depending on if it is rising or falling when moving from left to right.

Example 3:



$m = \frac{rise}{run} = \boxed{\frac{-8}{9}}$

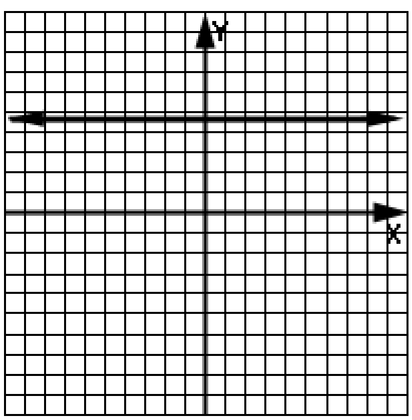
Example 4:



$m = \frac{4}{6} = \boxed{\frac{2}{3}}$

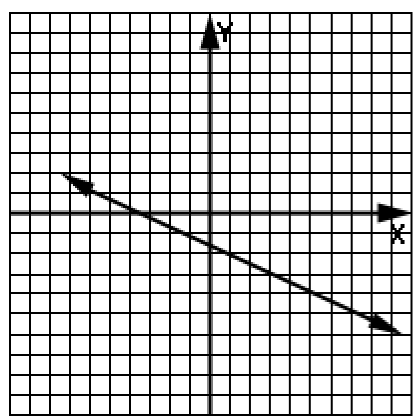
Third method: Just by looking at a line, moving from left to right, determine if the line is going up (positive slope), is going down (negative slope), is horizontal (zero slope), or is vertical (no slope, undefined).

Example 5:



0 slope

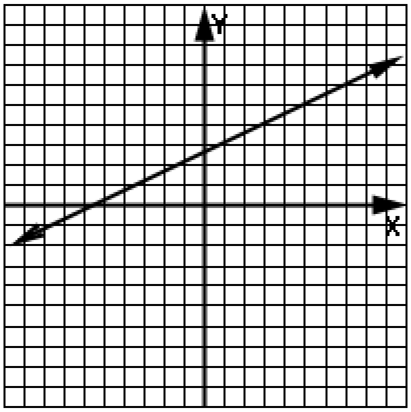
Example 6:



neg. slope

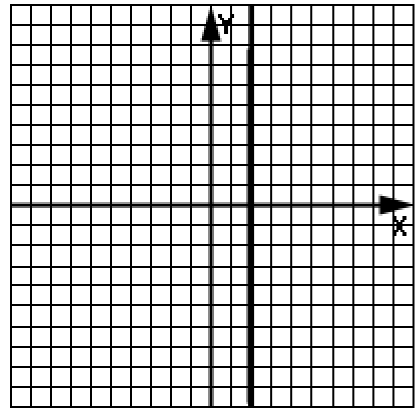
Example 7:

Positive Slope



Example 8:

Undefined, no slope



Fourth method: Convert the equation of a line to slope-intercept form (y = mx + b). The coefficient of *x* is the slope, *m*.

Example 9: What is the slope of the line $y = -13x + 5$?

$m = \boxed{-13}$

Example 10: What is the slope of the line $4x - 5y = 3$?

$-5y = -4x + 3$

$y = \frac{4}{5}x - \frac{3}{5}$

$m = \boxed{\frac{4}{5}}$

Slope relationships: If the slopes of two different lines are equal, the lines are parallel. If the slopes are negative reciprocals of each other (they multiply to give –1), the lines are perpendicular.

Example 11: Determine if the following two lines are parallel, perpendicular or neither:

y = 3x —1 and y — 3x = 5

$m_1 = 3$ $\quad y = 3x + 5$

$m_2 = 3$

slopes are the same, so they are parallel (//)

Example 12: Determine if the following two lines are parallel, perpendicular or neither:

y = 3x —1 and 3y = –x + 2

$m_1 = 3$ $\quad 3y = -x + 2$

$y = -\frac{1}{3}x + \frac{2}{3}$

$m_2 = -1/3$

$m_1 \cdot m_2 = 3(-1/3) = -1$, so they are perpendicular ($\perp$)

Assignment:

1. Find the slope of the line connecting the points (44, –1) and (–11, –6).

$x_1 y_1 \quad x_2 y_2$

$$m = \frac{-6-(-1)}{-11-44} = \frac{-5}{-55} = \boxed{\frac{1}{11}}$$

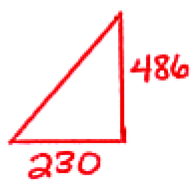
2. Find the slope of the line connecting the points (–1, –1) and (–1, –4).

$x_1 y_1 \quad x_2 y_2$

$$m = \frac{-4-(-1)}{-1-(-1)} = \frac{-4+1}{-1+1}$$

$$\frac{-3}{0} = \boxed{\text{No Slope undefined}}$$

3. A cone is 486 ft tall. From the bottom of the outside of the cone, the horizontal distance to the center of the cone is 230 ft. What is the slope of the cone?



$m = \frac{\text{rise}}{\text{run}}$

$m = \frac{486}{230}$

$m = \boxed{2.113}$

4. What is the slope of a vertical flagpole?

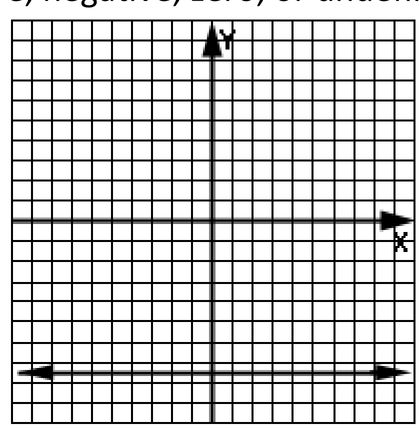
$\boxed{\text{undefined, no slope}}$

5. Find the slope of the line connecting the points (–4, 12) and (10, –6).

$x_1 y_1 \quad x_2 y_2$

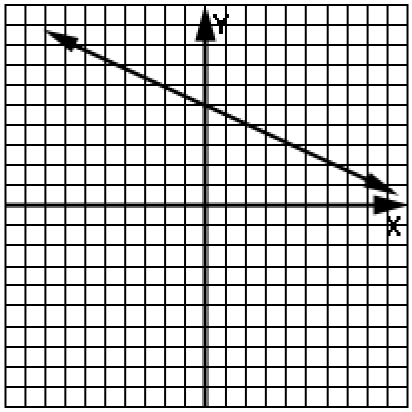
$$m = \frac{-6-12}{10-(-4)} = \frac{-18}{14} = \boxed{\frac{-9}{7}}$$

6. Identify the slope of this line as positive, negative, zero, or undefined.



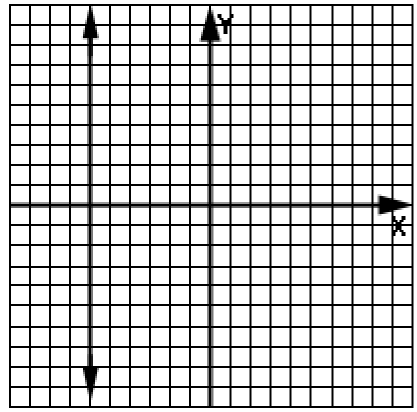
horizontal line, $m = 0$

7. Identify the slope of this line as positive, negative, zero, or undefined.



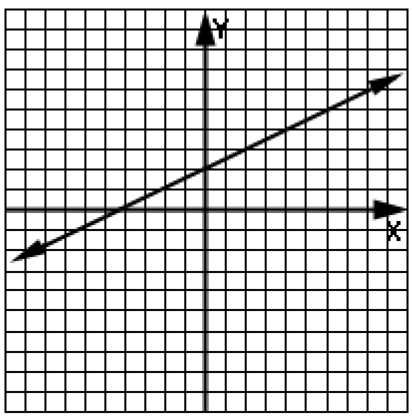
negative

8. Identify the slope of this line as positive, negative, zero, or undefined.



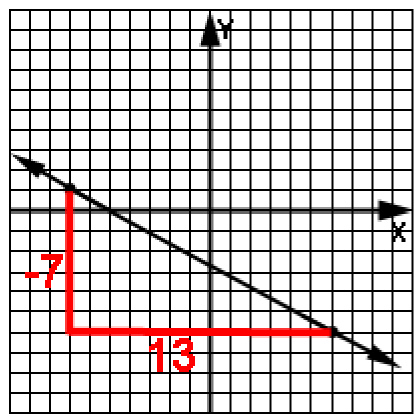
Vert line → no slope

9. Identify the slope of this line as positive, negative, zero, or undefined.



Positive

10. Use rise/run to determine the slope.



$m = \frac{rise}{run} = \boxed{\frac{-7}{13}}$

11. What is the slope of the line given by $3x - 15y = -1$?

$-15y = -3x - 1$

$y = \frac{-3}{-15}x - \frac{1}{-15}$

$y = \frac{1}{5}x + \frac{1}{15}$

$m = \boxed{\frac{1}{5}}$

12. What is the slope of the line given by $-11y = -4.5x - 2.2$?

$y = \frac{-4.5}{-11}x - \frac{2.2}{-11}$

$y = .4\overline{09}x + .2$

$m = \boxed{.4\overline{09}}$

13. What is the slope of the line given by $y = -4$?

$y = mx + b$

$y = 0x - 4$

$m = \boxed{0}$

14. What is the slope of the line given by $y = 13x - 2$?

$m = \boxed{13}$

15. What is the slope of the line given by $y = -21 - 5x$?

$y = -5x - 21$

$m = \boxed{-5}$

16. Find the slope of the line connecting the points (1,-2) and (-3, 4).

$x_1 y_1 \quad x_2 y_2$

$m = \dfrac{4-(-2)}{-3-1} = \dfrac{6}{-4} = \boxed{\dfrac{-3}{2}}$

17. Determine if the following two lines are parallel, perpendicular or neither:
y = 4x −2 and y − 4x = 5

$m_1 = 4 \qquad y = 4x + 5$

$m_2 = 4$

Slopes are the same

$\boxed{\text{parallel lines}}$

18. Determine if the following two lines are parallel, perpendicular or neither:
y = 3x −1 and the line connecting (1, 7) and (4, 6).

$y = 3x - 1 \qquad m_2 = \dfrac{6-7}{4-1} = \dfrac{-1}{3}$

$m_1 = 3$

$m_1 \cdot m_2 = 3\left(\frac{-1}{3}\right) = -1$

$\boxed{\text{They are } \perp}$

Unit 2: Lesson 02 Two forms for the equation of a line

Slope-intercept form: y= mx + b

m is the slope and **b** is the y-intercept (where line crosses y-axis)

Standard form: Ax + By = C

In both forms notice that x and y both are raised to the one power. Formally, we say they both are of degree one.

Special cases:

For a **horizontal line the equation is y = a number** that is the y-intercept.

For a **vertical line the equation is x = a number** that is the x-intercept.

(See **Enrichment Topic C** for two more forms of a line (point-slope & intercept).

In the following problems, use the provided information to write the slope-intercept form of the equation of the line.

Example 1: Slope is –2 and the y intercept is –5.

$m=-2 \quad b=-5$

$y=mx+b$

$\boxed{y=-2x-5}$

Example 2: Slope is 4 and the line passes through the point (11, –2).

$m=4$

$y=4x+b$ (Now sub in (11,-2))

$-2=4(11)+b$

$-2=44+b$

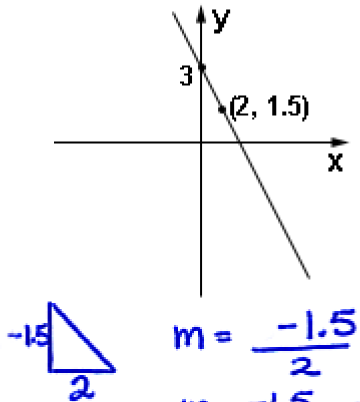
$-2-44=b$

$-46=b$

$y=mx+b$

$\boxed{y=4x-46}$

Example 3:



$m = \frac{-1.5}{2}$

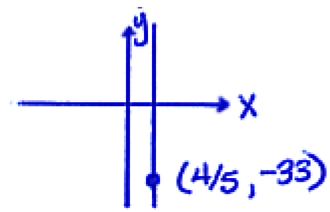
$m = \frac{-1.5}{2} = -.75$

$b = 3$

$y = mx + b$

$\boxed{y = -.75x + 3}$

Example 4: Slope is undefined and the line passes through the point (4/5, –33).



Vert. line, so equation is of the form $x = \#$

$\boxed{x = 4/5}$

Use a graphing calculator to graph two lines at the same time so as to compare the effect of different y-intercepts and slopes. (See **Calculator Appendix A** for instructions and a related video.)

Example 5: Slope is 2 and the x intercept is –4.

x int. is the point $(-4,0)$

$y = 2x + b$, now sub in $(-4,0)$

$0 = 2(-4) + b$

$0 = -8 + b$

$8 = b$

$y = mx + b$

$\boxed{y = 2x + 8}$

Example 6: The line passes through the points (1, –1) and (2, –7).

(x_1, y_1) (x_2, y_2)

$m = \frac{-7-(-1)}{2-1} = \frac{-6}{1} = -6$

$y = -6x + b$, now sub in $(1,-1)$

$-1 = -6(1) + b$

$-1 = -6 + b$

$-1 + 6 = b$

$5 = b$

$y = mx + b$

$\boxed{y = -6x + 5}$

Example 7: Parallel to the line given by $3x - 2y = 17$ and passing (1, 2).

$-2y = -3x + 17$

$y = \frac{3}{2}x - \frac{17}{2}$, So our line has a slope of 3/2.

$y = \frac{3}{2}x + b$, Now sub in (1,2)

$2 = \frac{3}{2}(1) + b$

$2 - \frac{3}{2} = b$

$\frac{1}{2} = b$

$y = mx + b$

$\boxed{y = \frac{3}{2}x + \frac{1}{2}}$

Example 8: Perpendicular to the line given by $y = .5x + 3$ and having a y intercept of 2.

$m = \frac{1}{2}$, neg. recip is

$m = -2$

$y = mx + b$

$\boxed{y = -2x + 2}$

Example 9: x-intercept 5 and y-intercept 6

$b = 6$ x int. (5,0)

$y = mx +$, sub in (5,0)

$0 = m(5) + 6$

$-6 = 5m$

$-\frac{6}{5} = m$

$y = mx + b$

$\boxed{y = -\frac{6}{5}x + 6}$

Example 10: Parallel to $x + y - 1 = 0$ and having a y intercept of 100.

$b = 100$

$x + y - 1 = 0$

$y = -x + 1$

$m = -1$

$y = mx + b$

$\boxed{y = -1x + 100}$

Example 11: Convert $y = -8x + 2$ to standard form.

$y = -8x + 2$

$\boxed{8x + y = 2}$

Example 12: Convert $44x - 2y = 9$ to slope-intercept form.

$44x - 2y = 9$

$-2y = -44x + 9$

$\boxed{y = 22x - \frac{9}{2}}$

Assignment:

In each problems 1-10 find the equation of the line.

1. Slope is –3 and the y intercept is – 4.

$m=-3 \quad b=-4$

$y=mx+b$

$\boxed{y=-3x-4}$

2. Slope is 2 and the line passes through the point (5, 6).

$m=2 \qquad y=mx+b$

$y=2x+b$

Sub in (5,6)

$6=2(5)+b$

$6=10+b$

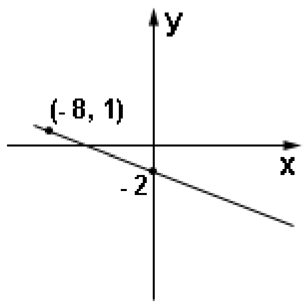
$6-10=b$

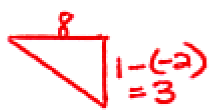
$-4=b$

$y=mx+b$

$\boxed{y=2x-4}$

3.



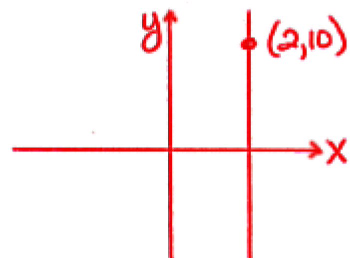


$m=\frac{\text{rise}}{\text{run}}=\frac{-3}{8}$

$b=-2$

$\boxed{y=-\frac{3}{8}x-2}$

4. Slope is undefined and the line passes through the point (2, 10).



Vert. line, eq is $x=\#$

$\boxed{x=2}$

5. Slope is 1 and the x intercept is –5.

$m=1 \qquad (-5,0)$

$y=mx+b$

$y=1x+b$, sub in $(-5,0)$

$0=1(-5)+b$

$5=b$

$y=mx+b$

$\boxed{y=1x+5}$

6. The line passes through the points (1, 0) and (–1, 7).

$(x_1,y_1) \qquad (x_2,y_2)$

$m=\frac{0-7}{1-(-1)}=\frac{-7}{2}$

$y=mx+b$

$y=-\frac{7}{2}x+b$, sub in $(1,0)$

$0=-\frac{7}{2}(1)+b$

$\frac{7}{2}=b$

$y=mx+b$

$\boxed{y=-\frac{7}{2}x+\frac{7}{2}}$

7. Parallel to the line given by $3x - 2y = 17$ and passing $(3, 9)$.

$-2y = -3x + 17$

$y = \frac{3}{2}x - \frac{17}{2}$

$m = \frac{3}{2}$

$y = mx + b$

$y = \frac{3}{2}x + b$, sub in $(3,9)$

$9 = \frac{3}{2}(3) + b$

$9 - \frac{9}{2} = b$

$\frac{9}{2} = b$

$y = mx + b$

$\boxed{y = \frac{3}{2}x + \frac{9}{2}}$

8. Perpendicular to the line given by $y = 2x + 3$ and having a y intercept of -1.

neg. recip of 2 is $-\frac{1}{2}$

$b = -1$

$y = mx + b$

$\boxed{y = -\frac{1}{2}x - 1}$

9: x-intercept 3 and y-intercept 8

$(3,0)$ $(0,8)$

$b = 8$

$y = mx + b$

$y = mx + 8$, sub in $(3,0)$

$0 = m(3) + 8$

$-8 = 3m$

$-\frac{8}{3} = m$

$y = mx + b$

$\boxed{y = -\frac{8}{3}x + 8}$

10. Parallel to $x + 7y - 1 = 0$ and having a y intercept of -20.

$x + 7y - 1 = 0$ $b = -20$

$7y = -x + 1$

$y = -\frac{1}{7}x + \frac{1}{7}$

$m = -\frac{1}{7}$

$y = mx + b$

$\boxed{y = -\frac{1}{7}x - 20}$

11. Convert $y = -6x + 5$ to standard form.

$y = -6x + 5$

$\boxed{6x + y = 5}$

12. Convert $28x - 2y = 8$ to slope-intercept form.

$28x - 2y = 8$

$-2y = -28x + 8$

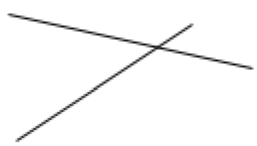
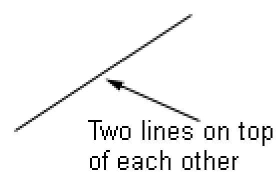
$\boxed{y = 14x - 4}$

Unit 2: Lesson 03 Graphical meaning of the solution to two linear equations

Consider the following two equations:

$$2x - 4y = 9$$
$$11x - 5y = -8$$

What does it mean to **solve this system of equations**? Very simply, it means to **find all the points of intersection** of these two lines. There are three distinct possibilities as shown below:

The two lines intersect in a single point. The *x* and *y* values of that point are the solutions to the system.	The two lines never intersect because the lines are parallel but separate.	The two lines are directly on top of each resulting in an infinite number of intersection points.
		

So, how can we tell by just looking at the equations of two lines which of the three pictures above represents their orientation?

If the **slopes of the two lines are different**, then it's the left picture above and we have only **one point** of intersection.

If the **slopes are the same** and the **y-intercepts are different**, then it's the middle picture above and we have **no points** of intersection.

If the **slopes are the same** and the **y-intercepts are the same** (they are actually the same line), then it's the right-hand picture above and there are **infinitely many points** of intersection.

In each example below, examine the slope and y-intercept. Then tell how many points are in the solution set of the system. Make a rough sketch of the lines.

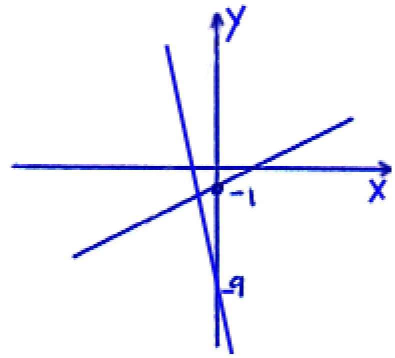
Example 1: $-4x = y + 9$ and $y = x - 1$

$-4x - 9 = y$

$m_1 = -4$ $m_2 = 1$

Slopes are different

(one intersection point)



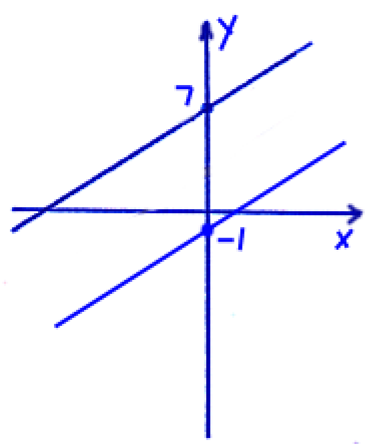
Example 2: $-x + y = 7$ and $y = x - 1$

$y = x + 7$ $m_2 = 1$

$m_1 = 1$

$m_1 = m_2$ so the lines are parallel. They have dif. b values so separate.

(No Solution)

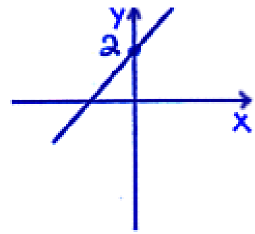


Example 3: $-x + y = 2$ and $6 + 3x - 3y = 0$

$y = x + 2$ $\quad$ $-3y = -3x - 6$

$m_1 = 1$ $\quad$ $y = x + 2$

$m_2 = 1$



Slope are the same; infact, the equations are the same. They sit on top of each other. So there are an infinite number of points as given by $y = x + 2$

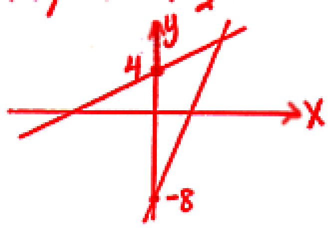
$\boxed{\text{Infinitely many solutions}}$

Assignment:

In the following problems, examine the slopes and y-intercepts of the two lines and then tell how many points are in the solution set of the system. Make a rough sketch of the lines.

1. $x + 4 = y \ and \ y = 3x - 8$

$m_1 = 1 \quad m_2 = 3$



Slopes are different
One solution point

---

2. $2x - 3y = -1$ and $8x - 12y = -4$

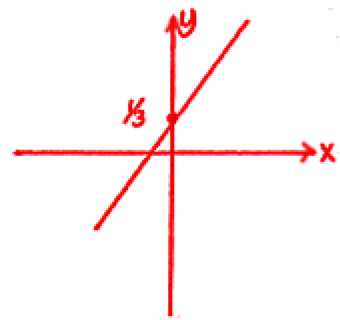
$-3y = -2x - 1$

$y = \frac{2}{3}x + \frac{1}{3}$

$-12y = -8x - 4$

$y = \frac{8}{12}x + \frac{4}{12}$

$y = \frac{2}{3}x + \frac{1}{3}$



Equations are the same
Infinite # of solution pts.
on the line $y = \frac{2}{3}x + \frac{1}{3}$

---

3. The line given by (1, –5) & (6, –10) and $3y = -3x + 11$

$m_1 = \frac{-10-(-5)}{6-1} = \frac{-5}{5} = -1$

$y = mx + b$ sub in pt. $(1, -5)$

$-5 = -1(1) + b$

$-5 + 1 = b$

$-4 = b$

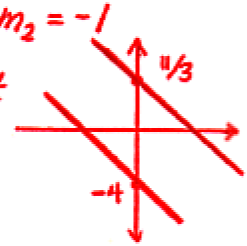
$y = \frac{-3}{3}x + \frac{11}{3}$

$y = -1x + \frac{11}{3}$

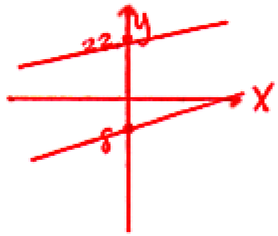
$m_2 = -1$

Slopes are =
while y int. are different
Lines are // & separate

$\boxed{\text{No Solution}}$



4. Line A has a slope of 3/5 and crosses the y-axis at –8. Line B has a slope of 3/5 and crosses the y-axis at 22.

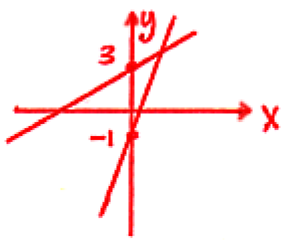


They are parallel & separate

$\boxed{\text{No Solution}}$

---

5. Line A has a slope of 1 and crosses the y-axis at 3. Line B has a slope of 3 and crosses the y-axis at –1.



$\boxed{\text{1 intersection point}}$

---

6. The product of the slope of the two lines is –1. Hint: This has something to do with the orientation of the line (parallel, perpendicular, etc).

$m_1 \cdot m_2 = -1 \longrightarrow$ they are perpendicular $\perp$,

$\boxed{\text{so they intersect in one point.}}$

---

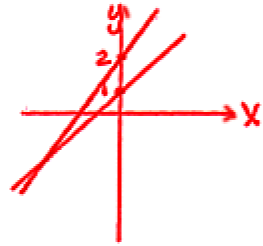
7. $y = 4.1x + 2 \quad and \quad 2y = 4.1x + 2$

$m_1 = 4.1 \qquad y = \frac{4.1}{2}x + 1$

$m_2 = \frac{4.1}{2}$

Slopes are different

$\boxed{\text{1 point of intersection}}$



8. The line connecting (0, 0) & (1,1) and the line connecting (–2, –2) & (5, 5)

$m_1 = \frac{1-0}{1-0} = 1$

$y = m_1x + b_1$

$0 = 1(0) + b_1$

$0 = b_1$

$y = m_1x + b_1$

$y = 1x + 0$

$m_2 = \frac{5-(-2)}{5-(-2)} = \frac{7}{7} = 1$

$y = m_2x + b_2$

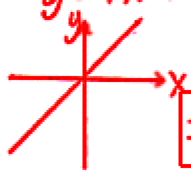
$5 = 1 \cdot 5 + b_2$

$0 = b_2$

$y = m_2x + b_2$

$y = 1x + 0$

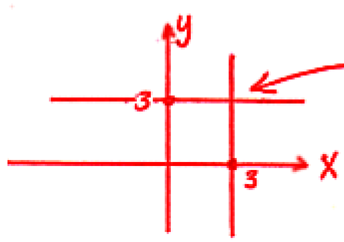
equations are identical



$\boxed{\text{Infinitely many pts on } y = x}$

---

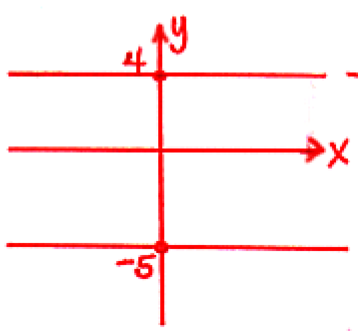
9. y = 3 and x = 3



$\boxed{\text{1 pt. of intersection}}$

---

10. y = 4 and y = –5



$\boxed{\text{Parallel \& separate. No solutions}}$

Unit 2: Lesson 04 — Algebraic solutions for two linear equations (elimination & substitution)

There are two primary algebraic ways to **find the intersection point** (the solution) of a system of two linear equations: **Substitution and elimination**

Substitution method:

Example 1: y = 3x – 2 and x + y = 6

$$x + (3x-2) = 6$$
$$x + 3x = 6+2$$
$$4x = 8$$
$$x = 2$$

$$y = 3x - 2$$
$$y = 3(2) - 2$$
$$= 6 - 2$$
$$y = 4$$

$$\boxed{x = 2 \quad y = 4}$$

Example 2: x + 2y = 9 and x + y = 4

$$x = 9 - 2y$$
$$9 - 2y + y = 4$$
$$9 - y = 4$$
$$-y = 4 - 9$$
$$-y = -5$$
$$y = 5$$

$$x = 9 - 2\cdot 5$$
$$x = 9 - 10$$
$$x = -1$$

$$\boxed{x = -1} \quad \boxed{y = 5}$$

Elimination method:

Example 3: 5x –2y = 8 and –3x + 2y =

$$5x - \cancel{2y} = 8$$
$$-3x + \cancel{2y} = 0$$
$$2x = 8$$
$$x = 4$$

$$-3\cdot 4 + 2y = 0$$
$$-12 + 2y = 0$$
$$2y = 12$$
$$y = 6$$

$$\boxed{x = 4} \quad \boxed{y = 6}$$

0

Example 4: 2x – 3y = 1 and 5x + 2y = 0

$$2(2x - 3y) = 1\cdot 2 \rightarrow 4x \cancel{- 6y} = 2$$
$$3(5x + 2y) = 0\cdot 3 \rightarrow 15x \cancel{+ 6y} = 0$$
$$19x = 2$$
$$\boxed{x = \frac{2}{19}}$$

$$2\left(\frac{2}{19}\right) - 3y = 1$$
$$\frac{4}{19} - 3y = 1$$
$$-3y = 1 - \frac{4}{19}$$
$$-3y = \frac{15}{19}$$
$$y = \frac{15}{19(-3)} = \boxed{\frac{-5}{19}}$$

Special cases:

Case 1:

If during the process of solving two linear equations, both variables cancel out and we are left with a **true** statement:

The interpretation is that we have **two lines right on top of each other**. There are infinitely many solutions and can be specified by giving the equation of either line.

Case 2:

If during the process of solving two linear equations, both variables cancel out and we are left with an **untrue** statement:

The interpretation is that we have **two parallel lines** that are separated by some distance and never meet. Therefore, there are **no solutions**.

Example 5:

$7x + y = 6$ and $2y = -14x + 12$

$-2(7x+y) = 6(-2) \rightarrow \cancel{-14x} \cancel{-2y} = \cancel{-12}$

$14x + 2y = 12 \rightarrow \underline{\cancel{14x} + \cancel{2y} = \cancel{12}}$

$0 = 0$

true

Equations are the same…
Infinitely many solutions:
All points on y = -7x + 6

Example 6:

$y = \left(\frac{3}{2}\right)x - 1$ and $2y = 3x + 8$

$y = \frac{3}{2}x - 1 \qquad 2y = 3x + 8$

$2\left(\frac{3}{2}x - 1\right) = 3x + 8$

$3x - 2 = 3x + 8$

$3x - 3x = 8 + 2$

$0 \neq 10$

Not true

$\boxed{\text{No solution}}$

Slopes are the same, but y-int are different… Two parallel & separate lines.

See **Enrichment Topic I** for how to solve three equations for three variables.

Assignment:

In the following problems, solve for the intersection point(s) of the systems of linear equations using the **substitution** method:

1. $y = 8x - 11$ and $x - 2y = 1$

$x - 2(8x - 11) = 1$

$x - 16x + 22 = 1$

$-15x + 22 = 1$

$-15x = 1 - 22$

$x = \frac{-21}{-15} = \boxed{\frac{7}{5}}$

$y = 8\left(\frac{7}{5}\right) - 11$

$y = \frac{56}{5} - \frac{11}{1}\left(\frac{5}{5}\right) = \frac{56}{5} - \frac{55}{5}$

$y = \boxed{\frac{1}{5}}$

2. $x = 4y + 4$ and $y = -3x + 1$

$y = -3(4y + 4) + 1$

$y = -12y - 12 + 1$

$y + 12y = -11$

$13y = -11$

$y = \boxed{\frac{-11}{13}}$

$x = 4\left(\frac{-11}{13}\right) + 4$

$x = \frac{-44}{13} + \frac{4}{1}\left(\frac{13}{13}\right)$

$x = \frac{-44}{13} + \frac{52}{13} = \frac{8}{13}$

$x = \boxed{\frac{8}{13}}$

3. $x + y = 6$ and $2y + 2x = 0$

$x = 6 - y$

$2y + 2(6 - y) = 0$

$2y + 12 - 2y = 0$

$0y + 12 = 0$

$12 \neq 0$

$\boxed{\text{No Solution}}$

4. $\left(\frac{1}{2}\right)y = \left(\frac{1}{3}\right)x + 2$ and $x - y - 1 = 0$

$6\left(\frac{1}{2}\right)y = 6\left(\frac{1}{3}\right)x + 6 \cdot 2$

$3y = 2x + 12$

$x = y + 1$

$3y = 2(y + 1) + 12$

$3y = 2y + 2 + 12$

$3y = 2y + 14$

$3y - 2y = 14$

$\boxed{y = 14}$

$x = 14 + 1$

$\boxed{x = 15}$

5. $x = 1$ and $y = x + 2$

$\boxed{x = 1}$

$y = 1 + 2$

$\boxed{y = 3}$

6. $18x - .5y = 7$ *and* $y = 8$

$18x - .5(8) = 7$

$18x - 4 = 7$

$18x = 7 + 4$

$18x = 11$

$\boxed{x = \frac{11}{18}}$ $\boxed{y = 8}$

In the following problems, solve for the intersection point(s) of the systems of linear equations using the **elimination** method:

7. $x + y = 5$ and $-x + 11y = 0$

$x + y = 5$
$-x + 11y = 0$
$12y = 5$
$\boxed{y = \frac{5}{12}}$

$-x + 11\left(\frac{5}{12}\right) = 0$
$-x + \frac{55}{12} = 0$
$-x = -\frac{55}{12}$
$\boxed{x = \frac{55}{12}}$

8. $4 = x + 2y$ and $-2y + x = 1$

$4 = 2y + x$
$2y + x = 4$

$\cancel{-2y} + x = 1$
$\cancel{2y} + x = 4$
$0 + 2x = 5$
$x = \boxed{\frac{5}{2}}$

$2y + x = 4$
$2y + \frac{5}{2} = 4$
$2y = 4 - \frac{5}{2} = \frac{8}{2} - \frac{5}{2} = \frac{3}{2}$
$y = \frac{3}{2 \cdot 2} = \boxed{\frac{3}{4}}$

9. $.4x + .2y = .1$ and $6x + 18y = 1$

$10(.4x + .2y) = 10(.1)$
$-9(4x + 2y) = 1(-9) \rightarrow -36x - \cancel{18y} = -9$
$6x + 18y = 1 \rightarrow 6x + \cancel{18y} = 1$
$-30x = -8$
$x = \boxed{4/15}$

$4x + 2y = 1$
$4\left(\frac{4}{15}\right) + 2y = 1$
$2y = 1 - \frac{16}{15}$
$2y = \frac{15}{15} - \frac{16}{15} = -\frac{1}{15}$
$y = -\frac{1}{15 \cdot 2} = \boxed{-\frac{1}{30}}$

10. $x + y + 1 = 0$ and $2x - 9y = 3$

$x + y = -1$
$-2(x + y) = -2(-1) \rightarrow -\cancel{2x} - 2y = 2$
$2x - 9y = 3 \rightarrow \cancel{2x} - 9y = 3$
$-11y = 5$
$y = \boxed{-\frac{5}{11}}$

$x + y = -1$
$x - \frac{5}{11} = -1$
$x = -1 + \frac{5}{11} = -\frac{11}{11} + \frac{5}{11}$
$= \boxed{-\frac{6}{11}}$

11. $x = 2$ *and* $x - 5y = 2$

$x - 5y = 2$
$-x \quad = -2$
$-5y = 0$

$\boxed{x = 2}$ $\boxed{y = 0}$

12. $y = 5x - 6$ and $-15x + 3y = -18$

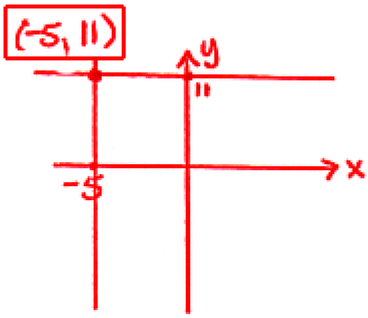
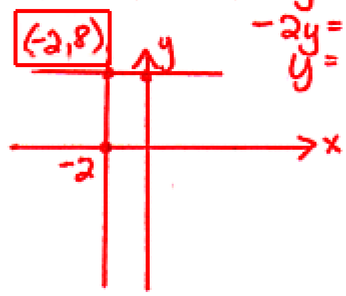
$-15x + 3y = -18$
$3y = 15x - 18$
$y = 5x - 6$

Same equation

Infinite # of pts. given by

$\boxed{y = 5x - 6}$

In the following problems, use any technique. Hint: Make a sketch of the two lines in which case you might be able to "see" the answer. This is called solving by "inspection."

13. $x = -5 \quad and \quad y = 11$ 	**14.** $x + 2 = 0 \quad and \quad 2(8 - y) = 0$ $x = -2$ $16 - 2y = 0$ $-2y = -16$ $y = 8$ 

Unit 2: Lesson 05 **Solving word problems with two linear equations**

Solve the following word problems by first defining the **two variables**. Then set up **two linear equations** in the two variables and solve by either substitution or elimination. Note that when we solve for **two variables** we must have **two equations**. Later, when we have three variables, we will need three equations, etc.

Example 1: Larry has $1.15 worth of coins in nickels and quarters. He has one more quarter than nickels. How many of each coin does he have?

N = # of Nickels
Q = # of Quarters

$.05N + .25Q = 1.15$
$Q = N + 1$

Solve by substitution

$.05N + .25(N+1) = 1.15$

$.05N + .25N + .25 = 1.15$

$.30N = 1.15 - .25$

$.30N = .90$

$N = \boxed{3}$

$Q = 3 + 1 = \boxed{4}$

3 Nickels, 4 Quarters

Example 2: Four years ago Bob was twice as old as his little sister. Five years from now he will be six years older than his sister. How old is each now?

B = Bob's age Now

S = Sister's age Now

$$\left.\begin{aligned} B-4&=2(S-4) \\ B+5&=(S+5)+6 \end{aligned}\right\} \text{2 equations}$$

$$B-4=2S-8$$
$$B-2S=-4$$

$$B+5=S+11$$
$$B-S=6$$
$$-2(B-S)=-2\cdot 6$$
$$-2B+\cancel{2S}=-12$$
$$B-\cancel{2S}=-4$$
$$-B=-16$$
$$\boxed{B=16}$$

$$16-S=6$$
$$-S=6-16$$
$$-S=-10$$
$$\boxed{S=10}$$

Assignment:

Solve the following word problems by first defining the two variables. Then set up two linear equations in the two variables and solve by either substitution or elimination.

1. The sum of two integers is 16 and their difference is 12. What are the numbers?

$x = 1^{st}$ int $= \boxed{14}$
$y = 2^{nd}$ int $= \boxed{2}$

$x + y = 16$
$x - y = 12$ } Two eq.
$2x = 28$
$x = 14$

$x + y = 16$
$14 + y = 16$
$y = 16 - 14 = 2$

---

2. An airplane flying into the wind can go 2000 miles in 6 hrs. On the return trip with the wind the flight time is 5 hr. What is the speed of the plane in still air and the speed of the wind?

p = speed of plane in still air
w = speed of wind

Dist = Rate · Time
with wind $2000 = (p+w)5 \rightarrow 2000 = 5p + 5w$
against wind $2000 = (p-w)6 \rightarrow 2000 = 6p - 6w$
p = plane speed
w = wind speed

$6 \cdot 2000 = 6(5p + 5w) \rightarrow 12000 = 30p + \cancel{30w}$
$5 \cdot 2000 = 5(6p - 6w) \rightarrow 10000 = 30p - \cancel{30w}$
$22{,}000 = 60p$
$\boxed{366.\overline{6} = p}$

$2000 = 5p + 5w$
$2000 = 5(366.6) + 5w$
$2000 = 1833.\overline{3} + 5w$
$2000 - 1833.\overline{3} = 5w$
$166.\overline{6} = 5w$
$\boxed{33.\overline{3} = w}$

3. Juan is 8 years older than his brother Two. In another 5 years, Juan will be twice as old as Two. How old are they now?

J = Juan's age now = $\boxed{11}$
T = Two's age now = $\boxed{3}$

$J = T+8$
$J+5 = 2(T+5)$

$J+5 = 2T+10$
$J = 2T+5$
(sub) $T+8 = 2T+5$
$T-2T = 5-8$
$-T = -3$
$T = 3$

$J = T+8$
$J = 3+8$
$J = 11$

4. A pile of 34 coins is worth $5.10. There are two nickels and the remainder are quarters and dimes. How many quarters and dimes are there?

Discard the two nickels and we have 32 coins and $5.00.

Q = Quarters
D = Dimes

$.25Q + .10D = 5.00$
(mult. both sides by 100 + get)
$25Q + 10D = 500$
$Q + D = 32$
2 equ.

$10(Q+D) = 32 \cdot 10$
$10Q + 10D = 320$
$-25Q - 10D = -500$

$-15Q = -180$
$Q = \boxed{12}$

$12 + D = 32$
$D = 32-12$
$D = \boxed{20}$

5. The 10's digit of a number is four more than the units digit of the number. The sum of the digits is 10. What is the number?

T = ten's digit
U = unit's digit

$T = U+4$
$T+U = 10$
$U+4+U = 10$
$2U = 10-4$
$2U = 6$
$U = \boxed{3}$

$T+U = 10$
$T+3 = 10$
$T = 10-3$
$T = \boxed{7}$

These, however, are not the answers to the question.

Original number = $10T + U = 10(7)+3 = \boxed{73}$
answer

6. The difference of two supplementary angles is 32 degrees. What are the angles?

x = 1st angle
y = 2nd angle

$x + y = 180$ (suppl.)
$x - y = 32$
$2x = 212$
$\boxed{x = 106}$

$x - y = 32$
$106 - y = 32$
$106 - 32 = y$
$\boxed{74 = y}$

7. Twins Jose and HoseB have a combined weight of 430 pounds. HoseB is 58 pounds heavier than Jose. How much does each weigh?

J = Jose's weight
H = HoseB's weight

$J + H = 430$
$H = J + 58$
$J + J + 58 = 430$
$2J = 430 - 58$
$2J = 372$
$\boxed{J = 186 \text{ lbs}}$

$H = J + 58$
$H = 186 + 58$
$\boxed{H = 244 \text{ lbs}}$

8. Angus earns $8.80 an hour at his Saturday job and $7.50 per hour at his after-school job. Last week he earned a total of $127.80. The hours he worked after school were 4 hours more than he worked on Saturday. How many hours did he work on Saturday?

x = Sat hrs
y = Week-day hrs

$y = x + 4$; $x(8.80) + y(7.50) = 127.80$
$8.80x + (x+4)(7.50) = 127.80$
$8.80x + 7.50x + 30 = 127.80$
$16.30x = 127.80 - 30$
$16.30x = 97.80$
$x = 97.80/16.30$
$x = \boxed{6 \text{ hrs}}$

Unit 2:
Lesson 06

Graphing calculator solutions of linear systems

Finding the intersection point of two lines:

Using the **Y=** button, enter the two linear equations as Y1 and Y2. Press the **Graph** button to display the two lines simultaneously.

If the intersection point of the lines is not visible, use the **Zoom** button and then zoom **In** or **Out**. The **ZStandard** zoom will often be the most useful. If none of these show the intersection point, make an estimate of where it is and use the **Window** button to adjust the max and min values accordingly.

With the intersection point displayed, access **2nd Calc | 5.intersect**. You will then be asked to identify the "first curve." Move the blinker with the left and right arrows until it is clearly on one of the lines. Press Enter. You will then be asked to similarly identify the "2nd curve." Finally, you are asked to "guess" the intersection. Move the blinker until it is very close to the intersection point and press Enter. The *x* and *y* values of the intersection point will be given at the bottom of the display.

See **Calculator Appendix C** and a related video for more details on the finding the intersection point of two lines.

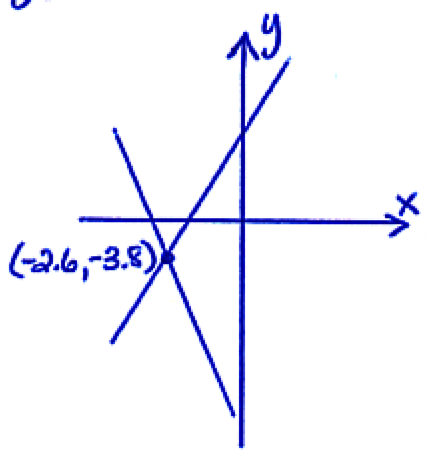
Using the techniques described above, find the intersection point of the following systems of linear equations. Make a rough sketch of the calculator display.

Example 1:

y= 3x + 4 and y= –2x – 9

$y_1 = 3x+4$

$y_2 = -2x-9$



Example 2:

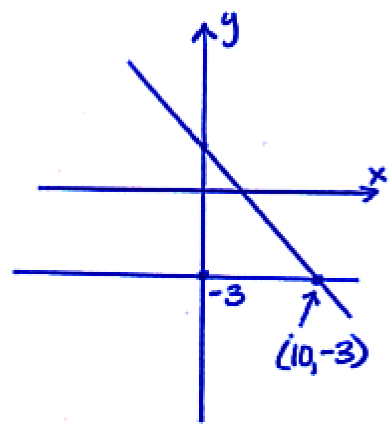
y = -3 and 2y – 4 + x = 0

$2y = -x+4$

$y = -\frac{1}{2}x+2$

$y_1 = -3$

$y_2 = -\frac{1}{2}x+2$

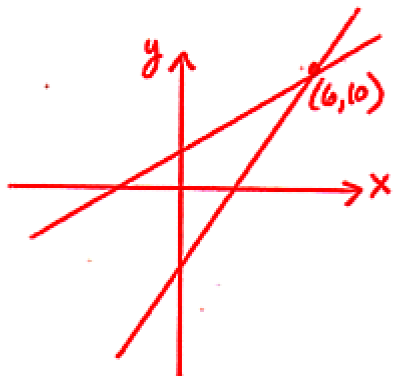


Assignment:

Using a graphing calculator, find the intersection point of the following systems of linear equations. Make a rough sketch of the calculator display.

1. $x + 4 = y$ *and* $y = 3x - 8$

$y_1 = x+4 \quad y_2 = 3x-8$

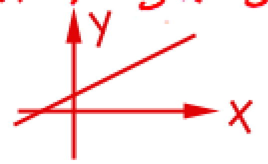


2. $2x - 3y = -1$ and $8x - 12y = -4$

$-3y = -2x - 1 \quad -12y = -8x - 4$

$y_1 = (-2x-1)/-3 \quad y_2 = (-8x-4)/-12$

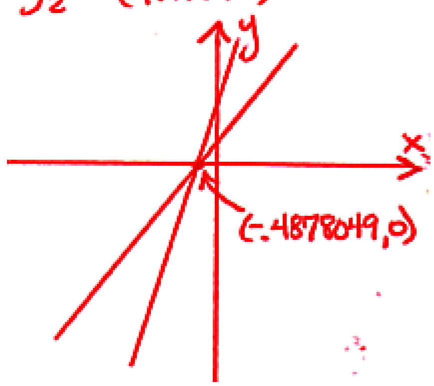
The two lines are right on top of each other. Infinite # of solutions on $y = \frac{2}{3}x + \frac{1}{3}$



3. $y = 4.1x + 2$ and $2y = 4.1x + 2$

$y_1 = 4.1x+2$

$y_2 = (4.1x+2)/2$

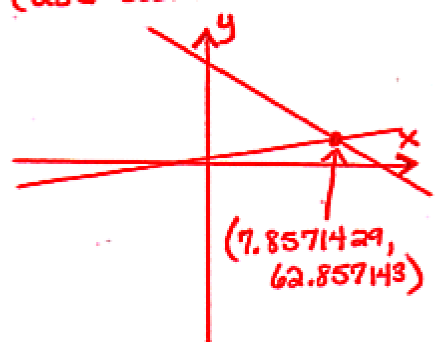


4. y = – 6x + 110 and y = x + 55

$y_1 = -6x+110$

$y_2 = x+55$

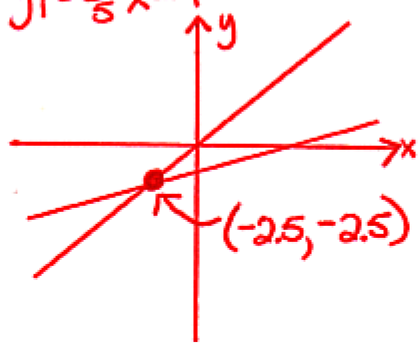
(use Zoom | Zoom Fit)



5. $\left(\frac{3}{5}\right)x - y = 1$ *and* $y = x$

$-y = \left(-\frac{3}{5}\right)x + 1 \quad y_2 = x$

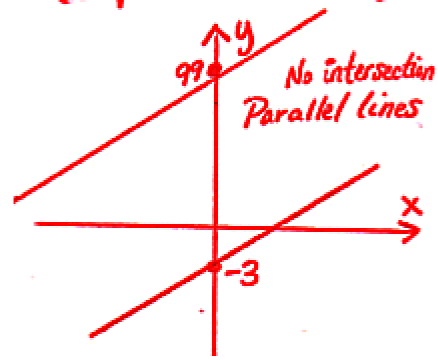
$y_1 = \frac{3}{5}x - 1$



6. y = x + 99 and y – x = -3

$y_1 = x + 99 \quad y_2 = x - 3$

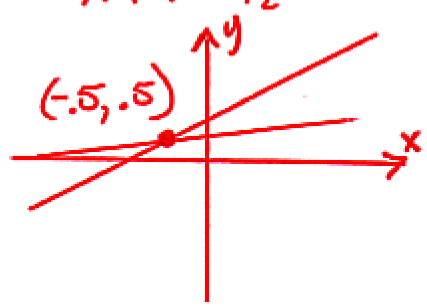
(slopes are the same)



7. .1 x + .2 = .3y and y = x + 1

$(.1x + .2)/.3 = y_1$

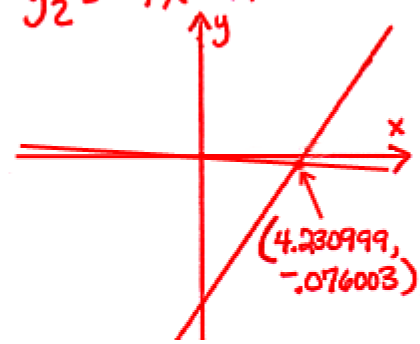
$x + 1 = y_2$



8. 1002y = –5x – 55 and y= 4x – 17

$y_1 = (-5x - 55)/1002$

$y_2 = 4x - 17$

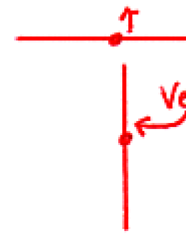


Unit 2: Review

1. What is the slope of the line containing the origin and (–3, –5)?

$m = \frac{-5-0}{-3-0} = \frac{-5}{-3} = \boxed{\frac{5}{3}}$

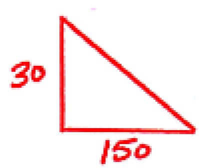
2. What is the slope of a horizontal line after being rotated 90 degrees?



Vertical line

$\boxed{\text{no slope undefined}}$

3. A hill falls 30 feet in altitude when moving from left to right. The horizontal distance covered is 150 feet. What is the “mathematical slope” of the hill?



$m = \frac{rise}{run}$

$m = \frac{-30}{150}$

$m = \boxed{-\frac{1}{5}}$

4. Convert y = –8x + 2 to general form.

$\boxed{8x + y = 2}$

5. What is the slope and y-intercept of the line given by 12x – 5y = 7?

$-5y = -12x + 7$

$y = \frac{12}{5}x - \frac{7}{5}$

$y = mx + b$

$m = \boxed{\frac{12}{5}}$ $b = \boxed{\frac{-7}{5}}$

6. How many points are there in the solution for the system, y = 5x – 8 and 3y –15x = 11?

$m_1 = 5$ $3y = 15x + 11$

$y = 5x + \frac{11}{3}$

$m_2 = 5$

Slopes are the same, so the lines are parallel. y-int. are different.

$\boxed{\text{No Points}}$

7. How many points are there in the solution for the system, y = 7x + 5 and y = –7x – 5 ?

One point

8. State whether the following two lines are parallel, perpendicular, or neither: y = 4x –1 and 4y =–x +2

$m_1 = 4$ $y = -\frac{1}{4}x + \frac{2}{4}$

$m_2 = -\frac{1}{4}$

$m_1 \cdot m_2 = 4(-\frac{1}{4}) = -1$

$\boxed{\text{Perpendicular}}$

9. Solve by elimination:

$5x - 2y = 8$ and $-3x + 2y = 2$

$$5x - 2y = 8$$
$$-3x + 2y = 2$$
$$2x = 10$$
$$x = \boxed{5}$$
$$5(5) - 2y = 8$$
$$25 - 2y = 8$$
$$-2y = 8 - 25 = -17$$
$$y = \boxed{\frac{17}{2}}$$

10. Solve by substitution:

$y = 3x - 4$ and $x - y = 6$

$$x - (3x - 4) = 6$$
$$x - 3x + 4 = 6$$
$$-2x = 6 - 4$$
$$-2x = 2$$
$$x = \boxed{-1}$$
$$y = 3(-1) - 4$$
$$y = -3 - 4 = \boxed{-7}$$

11. What is the intersection point of the horizontal line with y-intercept 6 and a vertical line with x-intercept –3?

$$\boxed{(-3, 6)}$$

12. Solve with substitution:

$-x + y = 4$ and $6x + 7y = 2$

$$y = x + 4$$
$$6x + 7(x + 4) = 2$$
$$6x + 7x + 28 = 2$$
$$13x = 2 - 28$$
$$13x = -26$$
$$x = \boxed{-2}$$
$$y = -2 + 4$$
$$y = \boxed{2}$$

13. The nerds in Saturday detention outnumbered the geeks by 6. Altogether there were 22 nerds and geeks. How many nerds and how many geeks were there? Define two variables and then set up two equations. Solve using either substitution or elimination.

N = # of Nerds

G = # of Geeks

$$N = G + 6$$
$$N + G = 22$$
$$(G + 6) + G = 22$$
$$2G = 22 - 6$$
$$2G = 16$$
$$G = \boxed{8}$$
$$N = 8 + 6$$
$$N = \boxed{14}$$

14. The smaller of two supplementary angles is exactly one-third the larger one. What are the two angles? Define two variables and then set up two equations. Solve using either substitution or elimination.

A = Smaller angle
L = Larger angle

$A = \frac{1}{3}L \qquad A + L = 180$

$\frac{1}{3}L + L = 180$

$\frac{4}{3}L = 180$

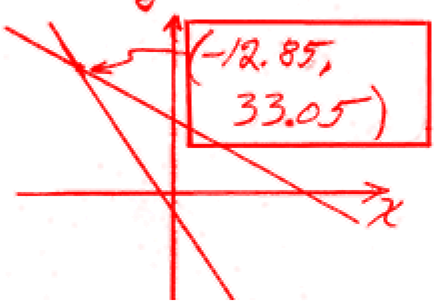
$L = 180\left(\frac{3}{4}\right) = \boxed{135}$

$A = \frac{1}{3}(135)$

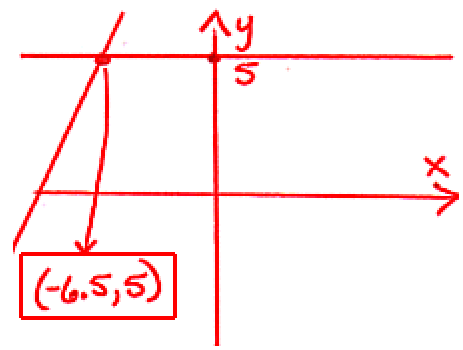
$A = \boxed{45}$

15. Using a graphing calculator, solve
$-13x - 5y = 1.8$ and $x + y = 20.2$

$-5y = 13x + 1.8$
$y1 = (13x + 1.8)/(-5)$
$x + y = 20.2$
$y2 = -x + 20.2$



16. Using a graphing calculator, solve
$y = 5$ and $y = x + 11.5$



Alg II, Unit 3

Graphing linear inequalities in two variables

Unit 3: Lesson 01 — Graphing a linear inequality in two variables

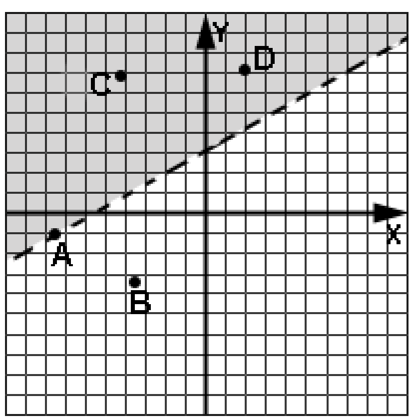
To graph an inequality like y < 3x – 5 , we first **draw the line** y = 3x – 5. Then do the following:

- If the inequality is **≥ or ≤** make the line **solid**. If the inequality is **< or >** make it **dotted**.
- If the inequality is **≤ or <**, shade **below** the line. If it is **≥ or >,** shade **above** the line. (This assumes *y* has been solved-for on the left.)
- If the line is vertical then **≤ or <** dictates that we shade to the left. Shade to the right if **≥ or >.**

  All the shaded points and/or a solid line are the solutions to the inequality.

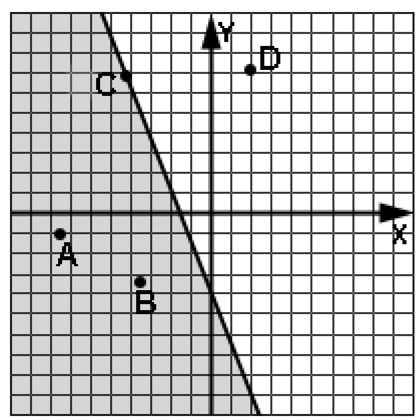
In Examples 1 and 2, identify those points that are solutions to the inequality.

Example 1:



C, D

Example 2:



A, B, C

In Examples 3 and 4, determine algebraically if the point is part of the solution to the inequality.

Example 3: 3x – 7y ≤ –2 (–4, 10)

$3(-4)-7(10)\le -2$

$-12-70\le -2$

$-82\le -2$ ✓

True, so the point (-4,10) is part of the solution. Yes!

Example 4: x < 2y – 17 (–8, 1)

$-8<2(1)-17$

$-8 \not< -15$

False!

(-8,1) is not part of the Solution. No!

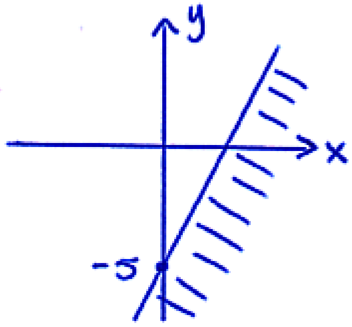
In examples 5 - 8, graph the inequality. Remember when dividing or multiplying by a negative number to reverse the inequality.

Example 5: $2x - y \geq 5$

$-y \geq -2x + 5$

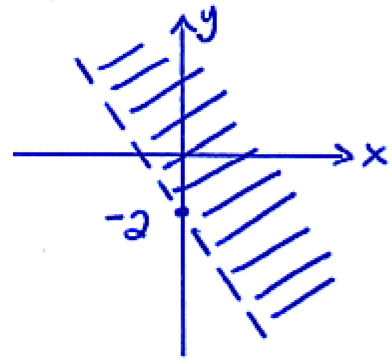
$y \leq 2x - 5$

Note reversal



Example 6: $x + y > -2$

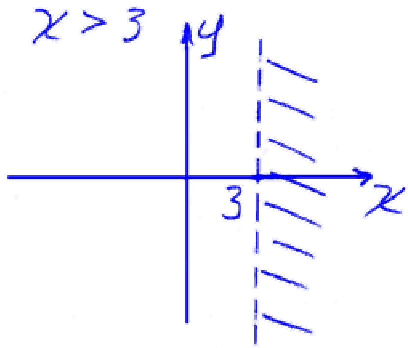
$y > -x - 2$



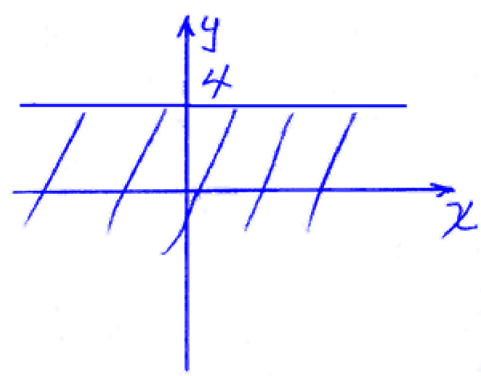
Example 7: $-x < -3$

$-x < -3$

$x > 3$



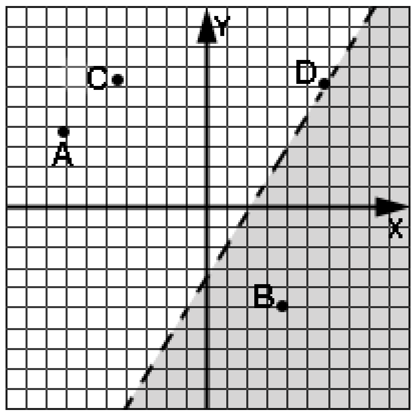
Example 8: $y \leq 4$



Assignment:

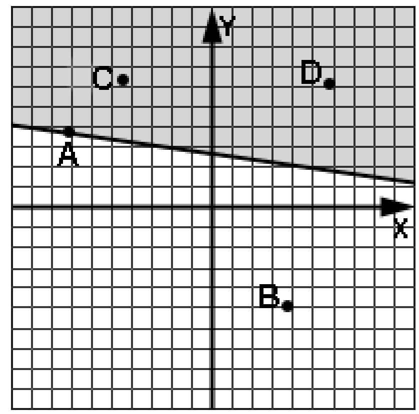
In problems 1 and 2, identify those points that are solutions to the inequality.

1.



B

2.



A, C, D

In problems 3 and 4, determine algebraically if the point is part of the solution to the inequality.

3. $77x - y < 2x - 1$ $(0, 0)$

$77(0) - 0 < 2(0) - 1$

$0 \not< -1$

False

(0,0) is not a solution.

No!

4. $10 \geq 4x - 7y$ $(-1, -2)$

$10 \geq 4(-1) - 7(-2)$

$10 \geq -4 + 14$

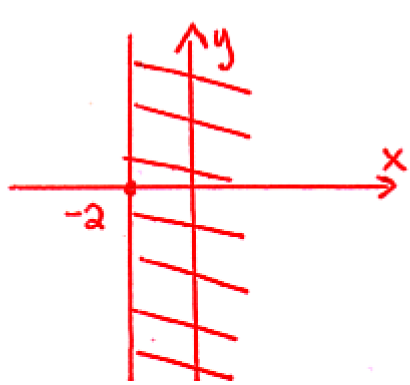
$10 \geq 10$

True!

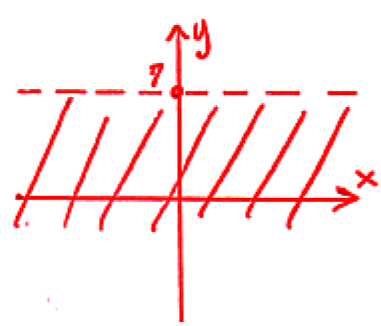
yes, (-1,-2) is a solution.

In problems 5 – 12, graph the inequality.

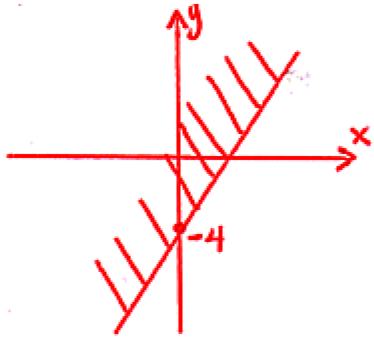
5. $x \geq -2$



6. $y < 7$



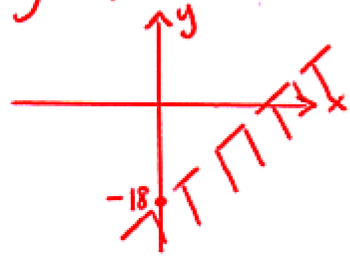
7. $y \geq 3x - 4$



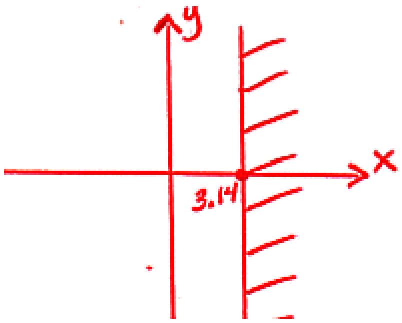
8. $x - y > 18$

$-y > -x + 18$

$y < x - 18$

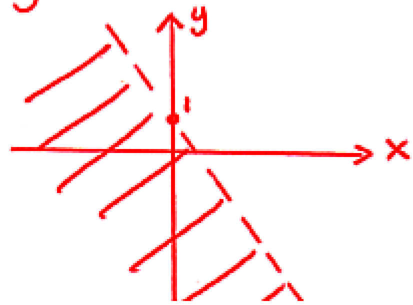


9. $x \geq \pi$



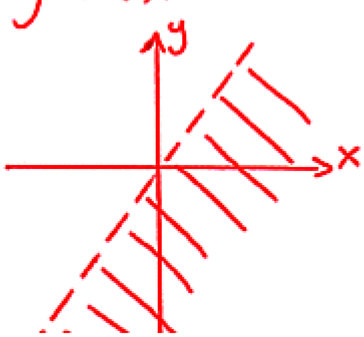
10. $x < -y + 1$

$y < -x + 1$

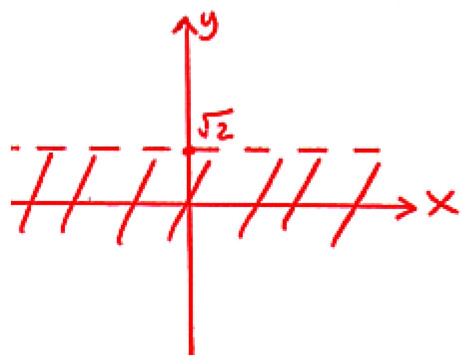


11. $3y < 12x$

$y < 4x$

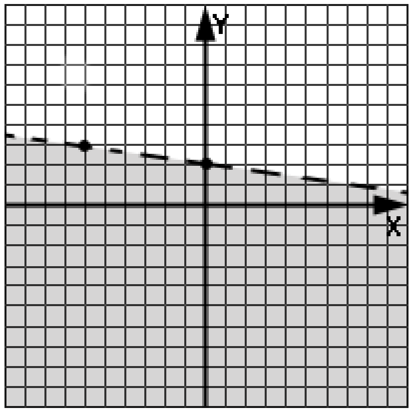


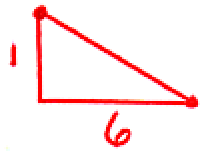
12. $y < \sqrt{2}$



In problems 13 and 14, state the inequality represented by the graph.

13.





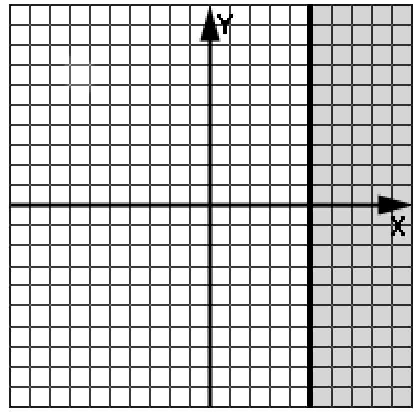
$m = -\frac{1}{6}$

$b = 2$

$y = mx + b$

$\boxed{y < -\frac{1}{6}x + 2}$

14.



$\boxed{x \geq 5}$

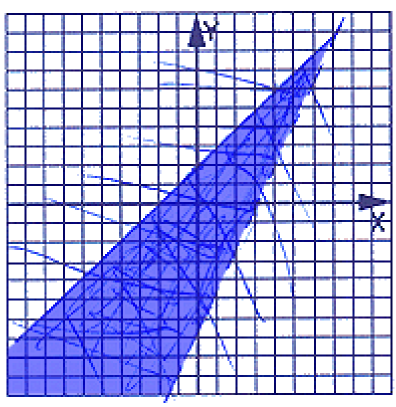
Unit 3: Lesson 02

Graphing systems of linear inequalities in two variables

In this lesson we will simultaneously graph more than one linear inequality in two variables. The solution to this “system” will be the region in which they all **intersect** (overlap).

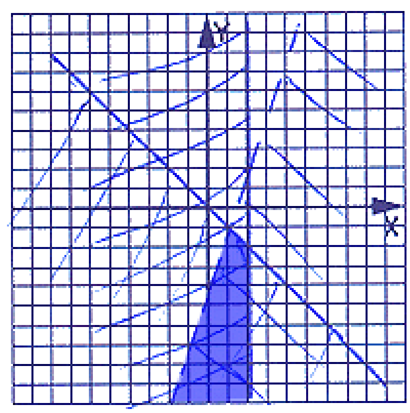
In the following examples, sketch the graph of the inequalities on the same coordinate system and then with heavier shading, show where they all intersect (overlap).

Example 1:



$y > 2x - 6$ and $y \le x + 2$

Example 2:



$3x - y > 4$, $x \le 2$, and $y \le -x$

$-y > -3x + 4$

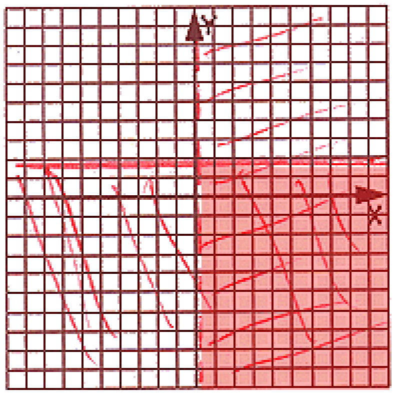
$y < 3x - 4$

See **Enrichment Topic B** for a “real world” application of linear inequality systems in two variables… **Linear Programming**.

Assignment:

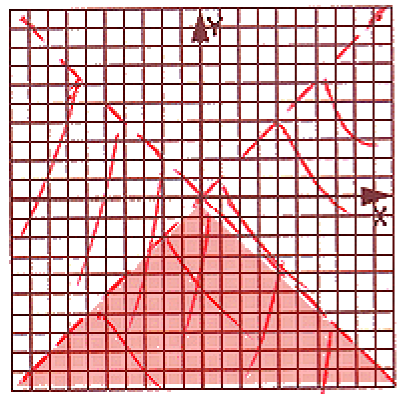
In the following problems, sketch the graph of the inequalities on the same coordinate system and then with heavier shading, show where they all intersect.

1.



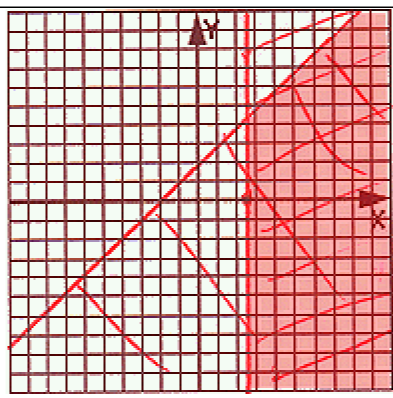
$x > 0, \quad y \le 2$

2.



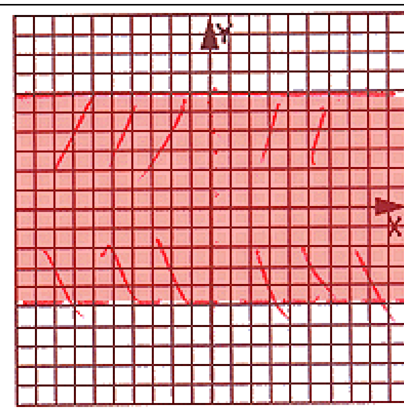
$y < x, \quad y < -x$

3.



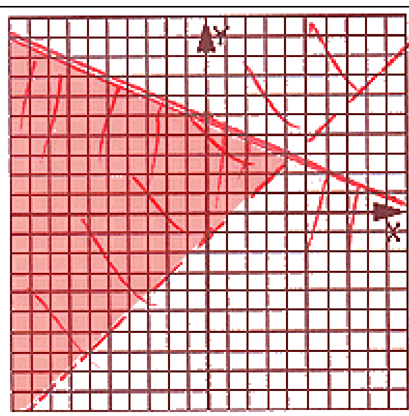
$y \le x + 2, \quad x \ge 2.5$

4.



$y > -5, \quad y \le 6$

5.

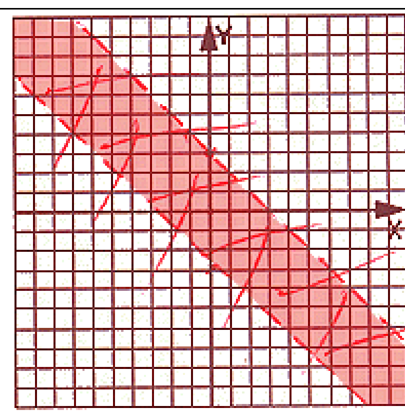


$x - y < 1, \quad 2y + x \le 10$

$-y < -x + 1 \quad 2y \le -x + 10$

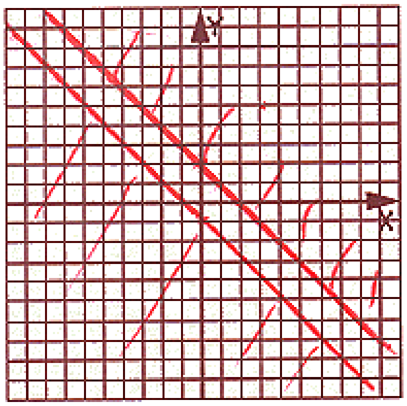
$y > x - 1 \quad y \le -\frac{1}{2}x + 5$

6.



$y < -x + 3, \quad y > -x - 3$

7.

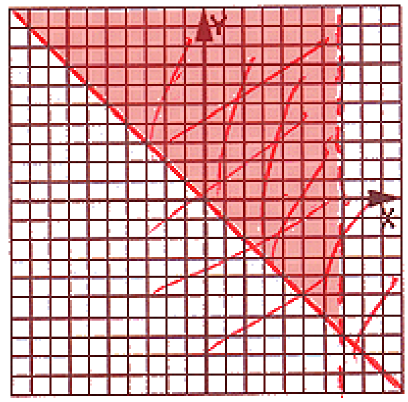


x + y + 1 ≤ 0,   –y ≤ x –2

$y \le -x - 1$ $y \ge -x + 2$

Parallel lines,
no intersection

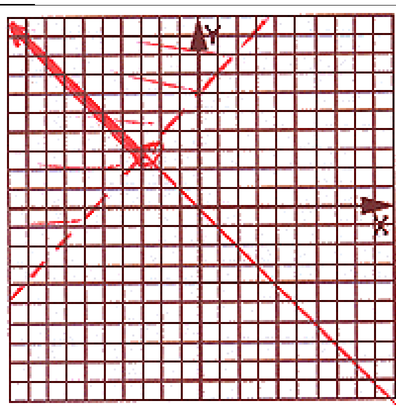
8.



x – 7 < 0,   x + y ≥ 0

$x < 7$ $y \ge -x$

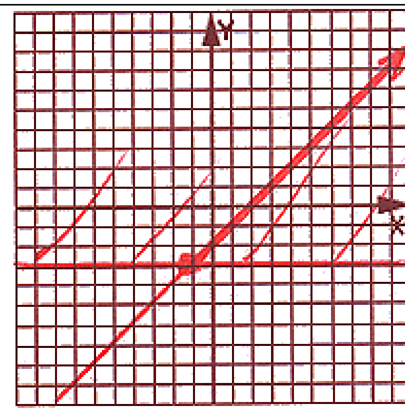
***9.**



y > x + 6,   y = –x

← answer

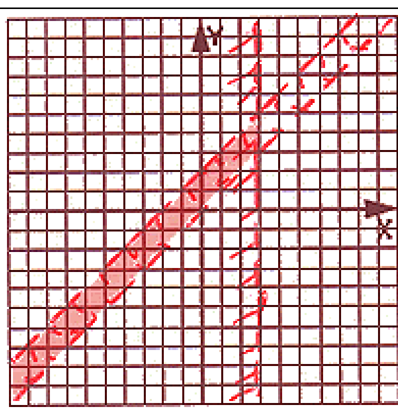
***10.**



y = x – 2,   y ≥ –3

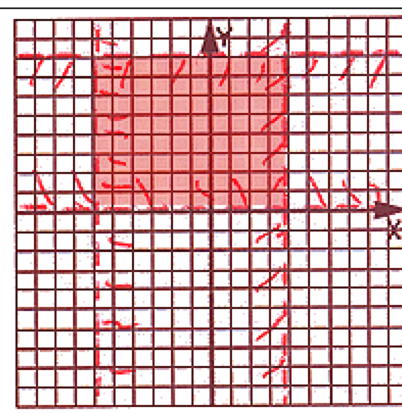
← answer

11.



y > x,   y < x + 2, and x < 3

12.



x < 4, x > –6, y > 0, and   y < 8

Unit 3: Lesson 03

*Graphing calculator- linear inequalities in two variables

The basics of graphing an inequality:
To graph an inequality in two variables on the calculator enter the function with the **Y=** button. After the function has been entered as **Y1**, use the left arrow to move the cursor to the left of **Y1**.

Repeatedly press **Enter** until a short diagonal line segment is displayed to the left of **Y1** with shading above the segment icon( ). This, in effect, converts **Y1 =** into **Y1 ≥**. Similarly, a short diagonal segment icon with shading below ( ) means ≤.

Select an appropriate zoom level (either with **Zoom** and/or **Window** buttons and then press **Graph**. The line will be drawn with the proper shading. You are on your own with regard to the difference between ≤ & < and ≥ & >. The calculator makes no distinction between them.

Multiple inequalities (Y1, Y2, etc.) can be displayed with this technique. The intersection representing the solution of the system will be evidenced by a dark cross-hatched region.

See **Calculator Appendix E** for more details.

How to graph a vertical line and associated inequality:
Strictly speaking, a vertical line cannot be graphed on a TI 84 or similar graphing calculator because a vertical line is multi-valued; therefore it is not a function (these calculators only graph functions). It is possible, however to closely **approximate a vertical line**.

> Suppose we need to graph the line $x = 5$ for Y2. The calculator won't allow a perfectly vertical line; however, we can approximate one with a very steep line passing through the point (5, 0). This would suggest the point slope form $y – 0 = m(x – 5)$.

Let the “very steep” slope, m, be -99999999999. Solving for y we get the following line that will become Y2 for the calculator.

Y2 = –9999999999(x – 5)

By using a negative slope this “vertical” line will be properly shaded to the left of Y2 using the technique described above (◣). In effect, this turns the line into x < 5.

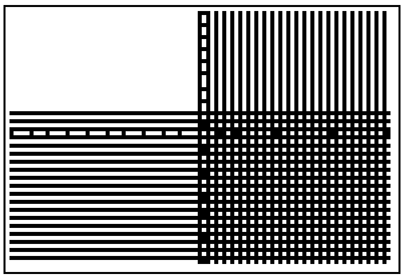
Similarly, with (◥), we are graphing x > 5 and the shading will be to the right.

See **Calculator Appendix B** for more details and a video on graphing vertical lines.

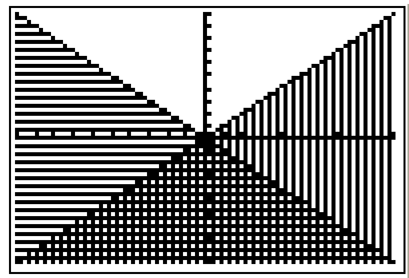
Assignment:

Use a graphing calculator to produce the solution set to the following systems of inequalities. Make a rough sketch of the answers. (These are the same problems from Lesson 2.)

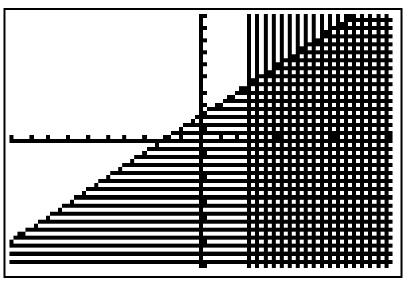
1. $x > 0, \quad y \le 2$



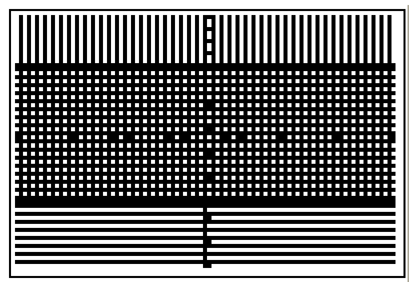
2. $y < x, \quad y < -x$



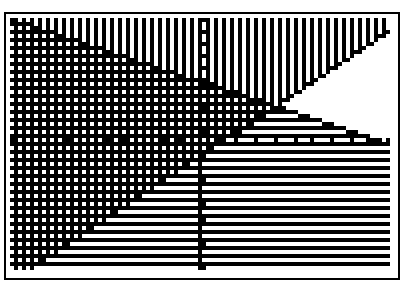
3. $y \le x + 2, \quad x \ge 2.5$



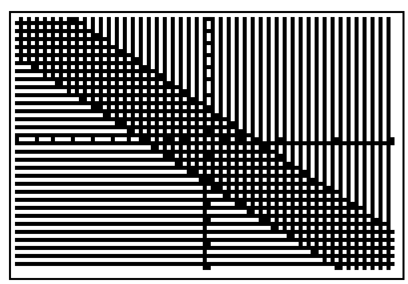
4. $y > -5, \quad y \le 6$



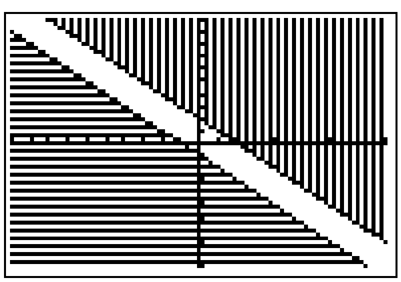
5. $x - y < 1, \quad 2y + x \le 10$



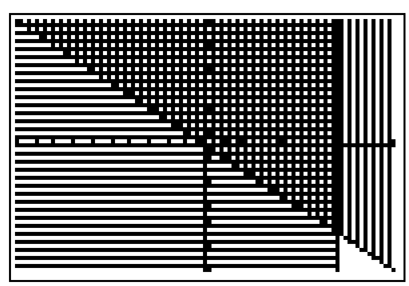
6. $y < -x + 3, \quad y > -x - 3$



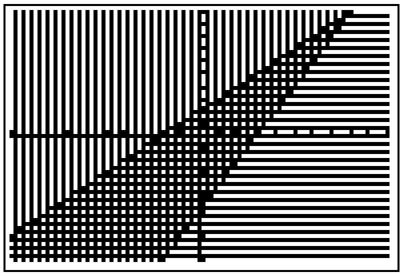
7. $x + y + 1 \le 0, \quad -y \le x - 2$



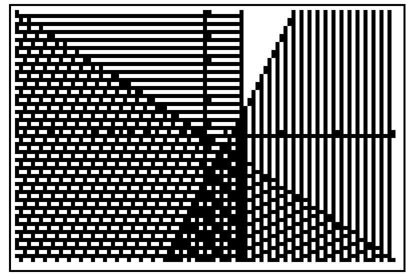
8. $x - 7 < 0, \quad x + y \ge 0$



9. y > 2x – 6 and y ≤ x + 2



10. 3x – y > 4, x ≤ 2, and y ≤ –x



Unit 3: Cumulative Review

1. Solve for x: $4x - x + 2 = 5 - x$

$3x + 2 = 5 - x$

$3x + x = 5 - 2$

$4x = 3$

$\boxed{x = 3/4}$

2. Solve $3 - x \geq 18$ and present the answer on a number line.

$3 - x \geq 18$

$-x \geq 18 - 3$

$-x \geq 15$

$\boxed{x \leq -15}$

-15 0

3. What is the slope of the line containing the points (–4, –2) and (6, –17)?

$x_1 y_1 \quad x_2 y_2$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-17-(-2)}{6-(-4)}$

$= \frac{-17+2}{6+4} = \frac{-15}{10} =$

$m = \boxed{-\frac{3}{2}}$

4. What is the equation of the line parallel to x + y = 17 and containing the point (–3, 1)?

$x + y = 17$

$y = -x + 17$

$m = -1$

$y = mx + b$

$y = -1x + b$

Sub in $(-3, 1)$

$1 = -1(-3) + b$

$1 = 3 + b$

$1 - 3 = b \quad b = -2$

$\boxed{y = -1x - 2}$

5. Find the point of intersection of the lines x + y = 5 and y = 6x – 1

$x + (6x - 1) = 5$

$7x - 1 = 5$

$7x = 5 + 1$

$7x = 6$

$x = \boxed{6/7}$

$y = 6x - 1$

$y = 6(6/7) - 1$

$y = \frac{36}{7} - 1$

$y = \frac{36}{7} - 7/7$

$y = \boxed{\frac{29}{7}}$

6. Determine if the lines 3x – y = 5 and –6x = 2y + 27 are parallel, perpendicular or neither.

$3x - y = 5$

$-y = -3x + 5$

$y = 3x - 5$

$m_1 = 3$

$-6x = 2y + 27$

$-2y = 6x + 27$

$y = -3x - \frac{27}{2}$

$m_2 = -3$

$\boxed{\text{Neither}}$

7. What is the relationship between the slopes of two lines that are perpendicular to each other?

One is the neg. reciprocal of the other.

or $(m_1)(m_2) = -1$

8. Solve $\frac{4x-1}{4} = -x$ for x.

$\cancel{4} \cdot \frac{4x-1}{\cancel{4}} = -x \cdot 4$

$4x - 1 = -4x$

$4x + 4x = 1$

$8x = 1$

$x = \boxed{1/8}$

Unit 3: Review

1. Consider the inequality $x < -4$. Which of these points are solutions to this inequality?
$(-6, 2)$, $(-4, -7)$, $(-1, 13)$, $(1, 4)$

$\boxed{(-6,2)}$

2. Consider the inequality $y \geq 2$. Which of these points are solutions to this inequality?
$(-8, 2)$, $(4, 7)$, $(0, 0)$, $(-2, -6)$

$\boxed{(-8,2),\ (4,7)}$

3. Determine algebraically if the point is included in the solution to the inequality.
$2x - 5y > 2 \quad (3, 1)$

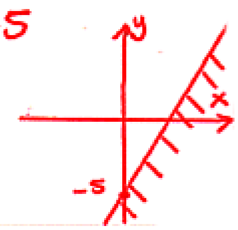
$2(3)-5(1) > 2$
$6 - 5 > 2$
$1 \not> 2$
No, not a solution

4. Determine algebraically if the point is included in the solution to the inequality.
$5x \leq y + 1 \quad (0, -1)$

$5(0) \leq -1+1$
$0 \leq 0$
yes, its part of the solution.

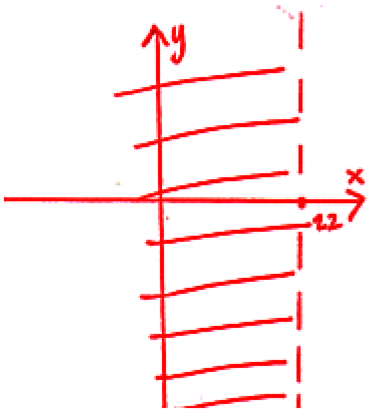
5. Graph the solution to $x - y \geq 5$.

$x-y \geq 5$
$-y \geq -x+5$
$y \leq x-5$

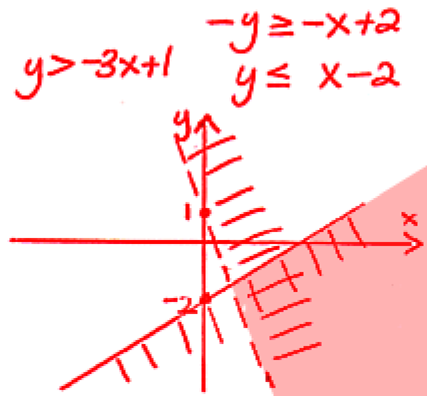


6. Graph the solution to $y < -x + 2$.

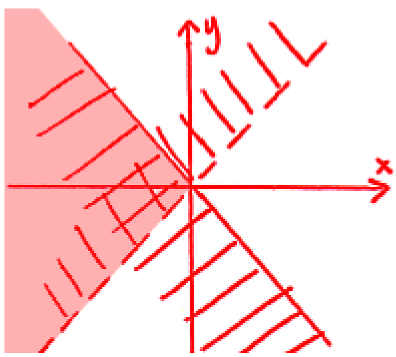
7. Graph the solution to x < 22.



8. Graph the solution to $x - y \geq 2$ and $-3x + 1 < y$.



9. Graph the solution to $y > x$ and $y \leq -x$.



10. Use a graphing calculator to show the solution to $x \leq 0$, $y \leq 0$, and $y > x$.