# Enrichment Topic D

## Combining direct and indirect variations

It is possible for a variable to be **proportional** to another variable while **simultaneously** being **inversely (indirectly) proportional** to yet another. The following example demonstrates this.

**Example 1:** The reds vary directly as the blues and inversely as the yellows. In a given case there are 12 reds, 3 blues, and 2 yellows. How many blues will there be when there are 14 reds and only 3 yellows?

$$R = \frac{kB}{Y}$$

$$RY = kB$$

$$\frac{RY}{B} = k$$

$$\frac{R_1 Y_1}{B_1} = k$$

$$\frac{R_2 Y_2}{B_2} = k$$

$$\frac{R_1 Y_1}{B} = \frac{R_2 Y_2}{B_2}$$

$$\frac{12 \cdot 2}{3} = \frac{14 \cdot 3}{B_2}$$

$$24 B_2 = 14 \cdot 9$$

$$B_2 = \frac{\overset{7}{\cancel{14}} \cdot 9}{\underset{12}{\cancel{24}}} = \frac{63}{12} = \boxed{\frac{21}{4}}$$

So, $\frac{R_1 Y_1}{B_1} = \frac{R_2 Y_2}{B_2}$

**Assignment:**

1. The number of yellows is directly proportional to the number of greens and inversely proportional to the number of whites squared. If the constant of proportionality is 14, write the yellows as a function of the greens and whites.

$$y = \frac{14G}{w^2}$$

2. Using the function developed in problem 1, how many whites would there be if there are 4 yellows and 6 greens?

$$y = \frac{14G}{w^2}$$

$$4 = \frac{14(6)}{w^2}$$

$$4w^2 = 84$$

$$w^2 = 21$$

$$w = \boxed{\pm\sqrt{21}}$$

3. The success of a particular business *S* is proportional to both the number of customers *N* and the quality of its products *Q*. Success is also inversely proportional to number of lazy workers *L*. Last year the success of the business was rated at 15. 1 while the number of customers was 1502, the quality of the products was rated at 5, and there were 23 lazy workers. What must be the quality number this year if the success number is to be 22 with only 1500 customers and 11 lazy workers?

$$S = \frac{kN \cdot Q}{L} \rightsquigarrow \frac{S_1 L_1}{N_1 Q_1} = \frac{S_2 L_2}{N_2 Q_2}$$

$$\frac{15.1(23)}{1502(5)} = \frac{22(11)}{1500\,Q_2}$$

$$15.1(23)(1500)Q_2 = 22(11)(1502)5$$

$$520950\,Q_2 = 1817420$$

$$Q_2 = \frac{1817420}{520950} = \boxed{3.489}$$

4. The relationship between *x* and *y* is $y = \frac{kx^3}{z}$. State this relationship in words using the words "directly", "inversely", etc.

y is directly proportional to the cube of X and inversely proportional to z.

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5. If *p* varies directly as *m*, directly as *q* squared, and inversely as *j*, what will be the new value of *m* when p = 6, q = 16, and j = 5? Formerly, p = 11, m= 10, q = 100, and j = 2.

$$p = \frac{kmq^2}{j} \rightsquigarrow \frac{p_1 j_1}{m_1 q_1^2} = \frac{p_2 j_2}{m_2 q_2^2}$$

$$\frac{11 \cdot 2}{10 \cdot 100^2} = \frac{6 \cdot 5}{m_2 \cdot 16^2}$$

$$m_2\, 22 \cdot 16^2 = 30 \cdot 10 \cdot 100^2$$

$$m_2\, 5632 = 3{,}000{,}000$$

$$m_2 = \boxed{532.67}$$