Unit 10:
Lesson 01   **Direct variation**

Consider **y = mx**.

There are several ways to describe the relationship between *x* and *y*:

- *y* varies directly as *x*
- *y* varies as *x*
- *y* is directly proportional to *x*
- *y* is proportional to *x*
- *y* varies linearly with *x*

When working with lines we call ***m*** the slope; however, in the context here we call it the **constant of proportionality**. In fact, most of the time we call the constant *k* and write:

$$y = kx$$

Solving for *k* we get:

$$\frac{y}{x} = k$$

Think of two ordered pairs that satisfy this equation, (x_1, y_1) and (x_2, y_2). Substituting these in we get two equations:

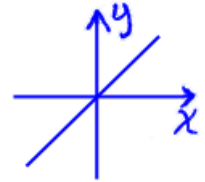$$\frac{y_1}{x_1} = k \quad \text{and} \quad \frac{y_2}{x_2} = k$$

Since both equal *k*, they equal each other:

$$\frac{y_1}{x_1} = \frac{y_2}{x_2}$$

Hence, ***y = kx*** implies $\dfrac{y_1}{x_1} = \dfrac{y_2}{x_2}$

Notice that when y is directly proportional to x

- the function y written as a function of x is a **linear function**,

- the constant of proportionality is the **slope** of the line,

- and the **y-intercept is 0**.

Example 1: y varies directly as x. When x is 3, y is 14. What is x when y is 11?

$$y = kx \longrightarrow \frac{y_1}{x_1} = \frac{y_2}{x_2}$$

$$\frac{14}{3} = \frac{11}{x_2}$$

$$14x_2 = 33$$

$$x_2 = \boxed{\frac{33}{14}}$$

Example 2: If b is directly proportional to x then b = 7 when x = 9. What is x when b = 11?

$$b = kx \longrightarrow \frac{b_1}{x_1} = \frac{b_2}{x_2}$$

$$\frac{7}{9} = \frac{11}{x}$$

$$7x = 99$$

$$x = \boxed{\frac{99}{7}}$$

Example 3: The profit p from the crop varies linearly with the amount of rainfall r. If p = \$62,000 when the rainfall is 26 inches, what is the expected profit when the rainfall is 32 inches?

$$p = kr \longrightarrow \frac{p_1}{r_1} = \frac{p_2}{r_2}$$

$$\frac{62,000}{26} = \frac{p}{32}$$

$$26p = (62,000)32$$

$$p = \frac{1,984,000}{26} = \boxed{\$76,307.69}$$

Example 4: p is directly proportional to q. When p is 10, q is 200. What is the constant of proportionality?

$$p = kq$$

$$10 = k\,200$$

$$\frac{10}{200} = k$$

$$\boxed{\frac{1}{20}} = k$$

Assignment:

1. c is proportional to j. When j is 2, c is 5. What is the value of j when c is 11?

2. When x is 9, p is 11. If x varies linearly with p, what is x when p is 10?

3. What is the constant of proportionality in problem 2?

4. Write the function describing the relationship between w and r when r varies as w.

5. The number of failures f was proportional to level of difficulty d of the material. The teacher found that 3 students failed when the difficulty level was 18. How many would fail if the difficulty level was raised to 36?

6. The reds vary linearly with the blues. If there are 14 reds when there are 4 blues, how many reds can be expected when there are only 2 blues?

7. z varies as m. Find the missing value in the table below.

m	z
2	6
14.1	?

8. If m varies as n with a constant of proportionality of 12, what will be the value of n when m = 104?

9. Sketch a graph of y as a function of x when it is known that y varies as x with a constant of proportionality of -2.

10. If y varies directly as x, what is the y-intercept when y is written as function of x and then graphed?

11. The population of a particular ancient civilization was always proportional to the amount of yearly rainfall. One year the population was 72,000 when the rain totaled 36 inches. What would be the population when the rain was only 20 inches?

12. Does the data in the table below indicate that there is a direct variation relationship between the two variables? (justify your answer.) If so, write the function rule.

x	y
3	9
5	15
8	24
11	33
15	45
32	96

13. Does the data in the table below indicate that there is a direct variation relationship between the two variables? (justify your answer.) If so, write the function rule.

x	y
2	4
6	12
7	14
12	36
20	60
30	120

14. Write the equation that indicates that j is proportional to d.

15. The constant of proportionality involved in the direct variation of b with c, is 1.2. What is the slope of the line resulting from graphing b as a function of c?

16. Consider the function relating Celsius temperature and Fahrenheit.

$$C = (5/9)(F - 32)$$

Does *C* vary directly as *F*? Justify your answer.

17. Which of the following equations illustrate direct variation?

    a. $y = 4x$
    b. $m + n = 0$
    c. $u = 3v + 2$
    d. $g = 4h + 0$
    e. $3y = 12x$
    f. $2y + 4x + 1 = 0$

18. If the number of healthy fish *f* varies as the amount *a* of food available, how many fish would be present when 6 lbs. of food are available? (Assume that 120 fish corresponds to 8 lbs. of food.)

19. What is the constant of variation if *y* varies directly as *x* and y = 5 when x = 20?

20. If *g* is proportional to *h* and the constant of variation is 22, what is the value of *g* when h = 11?