



Unit 10: Indirect variation

Lesson 02

Consider $y = k/x$.

There are two ways to describe the relationship between x and y :

- y varies inversely as x
- y is inversely proportional to x

Solve for k , the **constant of proportionality** and get:

$$\frac{y}{1} = \frac{k}{x} \quad xy = k$$

Substituting in two ordered pairs, (x_1, y_1) and (x_2, y_2) , that satisfy this equation, we get two equations:

$$x_1 y_1 = k \quad \text{and} \quad x_2 y_2 = k$$

$$\text{So } x_1 y_1 = x_2 y_2$$

Hence $y = k/x$ implies $y_1 x_1 = y_2 x_2$

Example 1: a is inversely proportional to b . When a is 9, b is 12. What is a when b has a value of 2?

$$a = \frac{k}{b} \rightarrow a_1 b_1 = a_2 b_2$$

$$9 \cdot 12 = a \cdot 2$$

$$\frac{9 \cdot 12}{2} = a$$

$$a = \boxed{54}$$

Example 2: The variable z varies inversely with h . What is the constant of proportionality when the pair $(h, z) = (5, -1)$?

$$z = \frac{k}{h}$$

$$-1 = \frac{k}{5}$$

$$-\frac{1}{1} = \frac{k}{5}$$

$$k = \boxed{-5}$$

Example 3: The average of the grades on a test is inversely proportional to the square of the amount of time spent in detention. What is the constant of proportionality when the test average is 75 and the detention time is 5 hours?

$$A = \frac{k}{T^2}$$

$$75 = \frac{k}{5^2}$$

$$75 = \frac{k}{25}$$

$$75(25) = k$$

$$\boxed{1875 = k}$$

Example 4: If y is inversely proportional to x and the constant of proportionality is 900, what is the value of x when y is 45?

$$y = \frac{900}{x}$$

$$45 = \frac{900}{x}$$

$$45x = 900$$

$$x = \boxed{20}$$

See Enrichment Topic D for how to solve problems that simultaneously combine direct and inverse (indirect) variation.

Assignment:

1. What is the constant of proportionality when y is inversely proportional to x , and $y = 4$ when $x = 10$?

2. Write the function describing the relationship between w and r when r varies inversely as w .

3. Write the function describing the relationship between w and r when r varies as w .

4. How many rabbits are there when there are 20 coyotes? The number of rabbits is inversely proportional to the number of coyotes where the constant of proportionality is 100.

5. z varies inversely as c . Find the missing value in the table below.

z	c
2	6
?	11

6. z varies inversely as m . Find the missing value in the table below.

m	z
2	6
14.1	?

7. T is inversely proportional to P . Find the missing data in the table below.

P	T
4	?
19	16

8. The number of free-throws f made was inversely proportional to the hostility h factor of the crowd. If 7 free throws were made when the hostility factor was 12, how many free-throws would be made when the hostility factor is only 4?

9. If m is directly proportional to n and $m = 12$ when $n = 2.6$, what would be the value of n when m is 21?

10. If j varies inversely with p , and the constant of proportionality is 7, what would be the value of p when $j = 5$?

11. Write the equation that indicates that A is inversely proportional to B.

12. If $g/h = 22$, is g directly proportional or inversely proportional to h ?

13. If the number n of a fish within a particular species varies inversely as the salinity of the water, and 200 fish are present when the salinity factor s is 15, how many fish will be present when the salinity is 20?

14. Does the data below indicate that there is an inverse variation relationship between the two variables? (Justify your answer.) If so, write the function rule.

x	y
4	25
5	20
10	11
20	6
25	5

15. Does the data below indicate that there is an inverse variation relationship between the two variables? (Justify your answer.) If so, write the function rule.

x	y
2	40
4	20
5	16
8	10
10	8

16. If the greens are inversely proportional to the reds, what is the constant of proportionality when there are 16 greens and 4 reds?

17. Using the information in problem 16, how many reds would there be if there are 48 greens?

18. Which of the following equations represent an inverse relationship between x and y ?

- a. $x + y = 1$ b. $3x - y = 0$ c. $x + 1/y = 0$ d. $y = 14/x$ e. $2/x = y$

19. Which of the choices of problem 18 represent a direct variation between the variables x and y ?