Unit 2: Lesson 04 Solving linear equations with variables on both sides

To solve an equation with **variables on both sides**, **eliminate** the variable on one side, thus collecting all of the variables on the other side.

It is not a requirement, but is suggested that variables be collected on the **left side** of the equation.

Example 1: Solve 4x – 6 = 7x

$$4x - 6 = \cancel{7x}$$
$$-7x \qquad \cancel{-7x}$$
$$-3x \cancel{-6} = 0$$
$$\cancel{6} \qquad 6$$
$$-3x = 6$$

$$\frac{\cancel{-3}x}{\cancel{-3}} = \frac{6}{-3}$$
$$x = \boxed{-2}$$

At this point we want to increase our level of sophistication in how to add (or subtract) numbers or terms from each side of an equation.

In the following example (which is the same problem in Example 1), notice how we still add –7x and +6 to both side, but in a new way.

Example 2: Solve 4x – 6 = 7x

$$4x - 6 = 7x$$
$$4x - 6 - 7x = \cancel{7x - 7x}$$
$$-3x - 6 = 0$$
$$-3x \cancel{-6+6} = +6$$
$$-3x = 6$$

$$\frac{\cancel{-3}x}{\cancel{-3}} = \frac{6}{-3}$$
$$x = \boxed{-2}$$

Example 3: Solve 4(2 + x) – 5x = x + 12

$$4(2+x)-5x=x+12$$
$$8+\underbrace{4x-5x}=x+12$$
$$8-x=x+12$$
$$8-\underbrace{x-x}=\underbrace{\cancel{x}+12-\cancel{x}}$$
$$8-2x=12$$
$$\underbrace{\cancel{8}-2x-\cancel{8}}=12-8$$
$$-2x=4$$
$$\frac{\cancel{-2}x}{\cancel{-2}}=\frac{4}{-2}$$
$$x=\boxed{-2}$$

Example 4: Solve 2(y – 3) + 4 = 6(7 – y)

$$2(y-3)+4=6(7-y)$$
$$2y-\underbrace{6+4}=42-6y$$
$$\underbrace{2y-2+6y}=42\cancel{-6y}\cancel{+6y}$$
$$8y-2=42$$
$$8y\cancel{-2}\cancel{+2}=42+2$$
$$8y=44$$
$$\frac{\cancel{8}y}{\cancel{8}}=\frac{44}{8}$$
$$y=\frac{44}{8}=\boxed{\frac{11}{2}}$$

Example 5: Solve 2(f –3) = 2(f – 2) – 5

$$2(f-3) = 2(f-2) - 5$$

$$2f - 6 = 2f - 4 - 5$$

$$\cancel{2f} - 6 - \cancel{2f} = \cancel{2f} - 9 - \cancel{2f}$$

$$-6 \neq -9$$

No solution

Sometimes (as in the example above) a statement is produced that is not true. This means there is **no solution** to the equation.

Example 6: Solve 4(z + 5) – 8 = 4(z + 3)

$$4(z+5) - 8 = 4(z+3)$$

$$4z + 20 - 8 = 4z + 12$$

$$\cancel{4z} + 12 - \cancel{4z} = \cancel{4z} + 12 - \cancel{4z}$$

$$12 = 12$$

All real x

ARX

Sometimes (as in the example above) a statement is produced that is true; however, the variables are no longer present (they all canceled out). This means there are an **infinite number of solutions** (all real numbers).

Assignment: Solve the following equations.

1. $3(x + 6) = 5(x + 2)$

2. $v + 8 = 5v + 5(1 - v)$

3. $6x - 2 + x = 7x - 13$

4. $-11d - 2d - 1 = 27 + d$

5. $-3(p + 5) + 6 = 3(-p - 3)$

6. $-4m - 9 + 5m = 51 - 5m$

7. $w - 4(w + 2) = 7 - 2w$

*8. $(3/2)x + 1/2 = (7/3)x + 4$

9. $3q - (2 - q) = 2(2q - 1)$

10. $5r + 22r - 7 = 2 - r$

11. $8(j + 2) + 9(-j - 1) = -j + 2$

*12. $2[\ x - 3(-x - 5) + 1\ ] = 2(\ x + 11) - (-4 + x)$