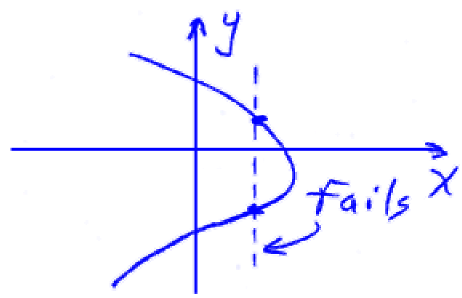
Unit 5: Lesson 03 Functions, function notation

Graphical function definition:

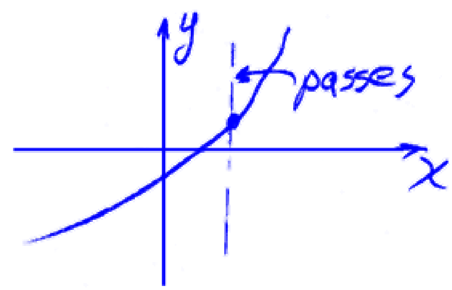
A function is a relation that passes the **vertical line test**:

When a vertical line is drawn anywhere on a graph of the relation, it must touch no more than one point of the relation.

Example 1: Draw an example of a relation that does not pass the vertical line test.



Example 2: Draw an example of a relation that passes the vertical line test.



What about relations that are not given as graphs (ordered pair lists, tables, or mappings)? How do we determine if they "pass the vertical line test?"

Abstract function definition:

For a relation to be a function, no two first-coordinates (the *x* values) can be the same.

Example 3: Is the relation given by { (3, 5), (8, -9), (3, 14) } a function? Why?

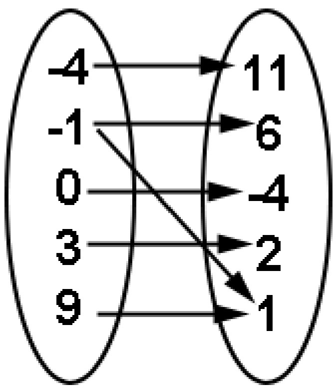
No, it is not a function. The x-coordinate, 3, is repeated.

Example 4: Is the relation given by this table a function? Why?

x	y
5	6
-3	11
8	-9
0	11
2	22

Yes, it is a function. All the x-coordinates are different.

Example 5: Is the relation given by this mapping a function? Why?



No, it is not a function. The ordered pairs this mapping represents are { (-4, 11), (-1, 6), (-1, 1), (0, -4), (3, 2), (9, 1) }.

Notice that the x-coordinate, -1, is repeated.

A function can be represented as a **rule** (a mathematical formula):

Think of the rule as a factory and as a given domain (the *x's*) as the raw material input to the factory. The factory (the rule) then produces *y* values (the values of the range).

Example 6: Given the function rule, y = 3x + 1, whose domain is { -2, 1, 5, 7 }, produce the corresponding range.

X (input)	Y (output)
-2	**-5**
1	**4**
5	**16**
7	**22**

$y = 3(-2)+1 = -6+1 = -5$

$y = 3(1)+1 = 3+1 = 4$

$y = 3(5)+1 = 15+1 = 16$

$y = 3(7)+1 = 21+1 = 22$

$\boxed{\text{Range: } \{-5, 4, 16, 22\}}$

Function notation:

One way to write a function rule is, for example, y = -2x + 6.

Another way to write the same rule is, f(x) = -2x + 6.

Read f(x) as **"f of *x*"**; however, just think of f(x) as meaning y.

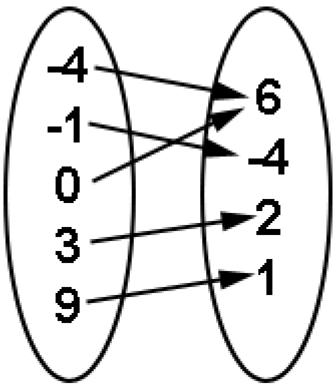
When we see something like f(-5), this just means to replace all the *x's* in the function rule with -5 and then simplify.

Example 7: Evaluate f(-4) when f(x) = 6x + 3 – x.

$$f(x) = 6x + 3 - x$$
$$f(-4) = 6(-4) + 3 - (-4)$$
$$= -24 + 3 + 4$$
$$= -21 + 4 = \boxed{-17}$$

Assignment: In problems 1-8, decide if the given relation is a function or not.
Justify your answer.

1.



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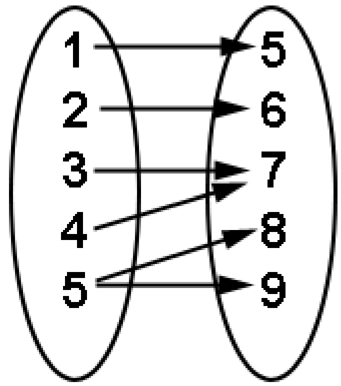
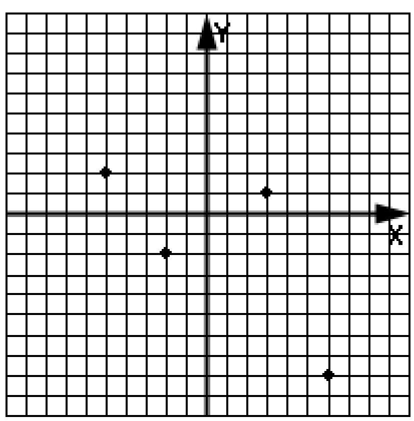


3. { (11, 2), (–4, 13), (11, –1), (3, 18) }

4. { (12, –2), (–4, –2), (11, –2), (16, –2) }

5.



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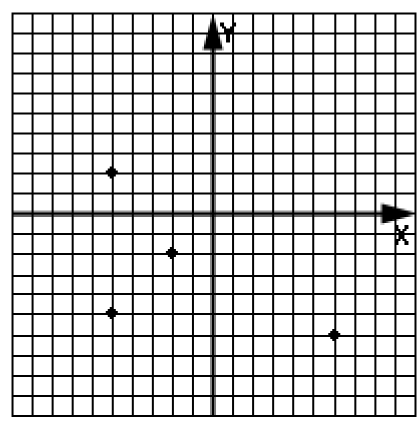

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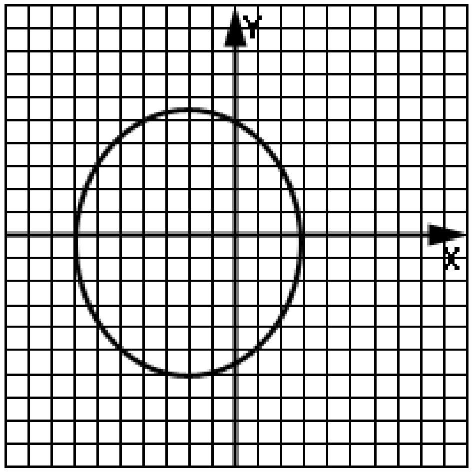

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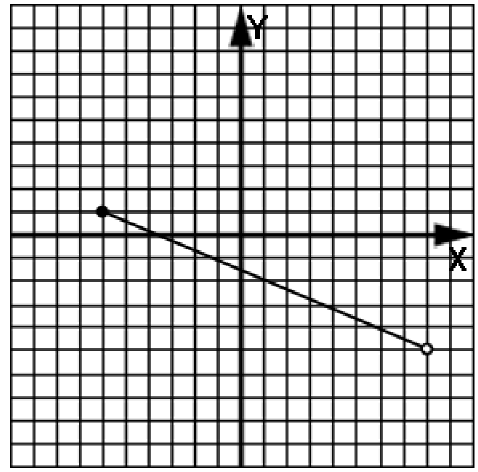

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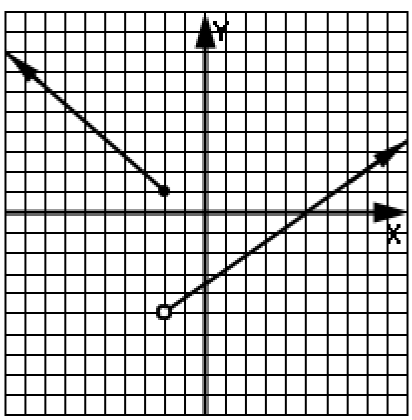
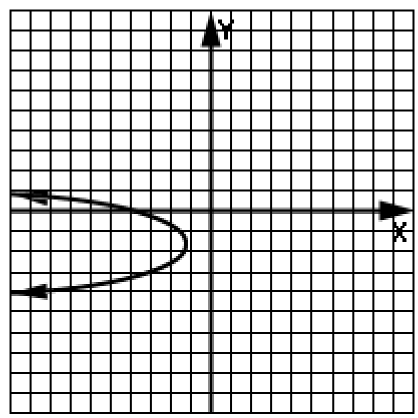


10.



11.

X (input)	**f(x) (output)**
-3	-5
9	4
-3	11
7	23

12.

X (input)	**f(x) (output)**
-3	8
9	8
-4	11
7	8

13. Find f(11) when $f(x) = (x + 3)/ (x – 22)$.

14. Given the function rule $f(x) = x + 9$, find the range corresponding to the domain, { -3, 4, 6, 8 }. Fill in the table below with the domain and range.

X (input)	**f(x) (output)**

15. Find the range for $f(x) = x^2 + 3$ if its domain is { -4, 6, 10 }.

16. Give the domain and range for the relation in problem 1.

17. Give the domain and range for the relation in problem 2.

18. Give the domain and range for the relation in problem 6.

19. Give the domain and range for the relation in problem 12.

20. Evaluate $f(x) = x^2 + 3x + 1$ at $x = -2$.