



Unit 5: Function word problems
Lesson 05 Constant rates of change

When a quantity is changing at a constant rate (either increasing or decreasing) the quantity at any time t is given by the function $Q(t)$:

$$Q(t) = b + \text{rate}(t) \quad \text{where } b \text{ is the starting quantity and } \textit{rate} \text{ is the rate of change.}$$

Example 1: Larry Large currently weighs 380 pounds. He plans to go on a diet and lose 4 lbs per week. What is the starting quantity? What is the rate? Write his weight as a function of time, $w(t)$.

t, weeks	w(t), lbs
0	$380 - 4(0) = 380$
1	$380 - 4(1) = 376$
2	$380 - 4(2) = 372$
3	$380 - 4(3) = 368$

starting quantity = 380 lbs
rate = -4 lbs/week

$$w(t) = 380 - 4t$$

Example 2: After how many weeks will Larry Large weigh 300 lbs?

$$\begin{aligned}
 w(t) &= 380 - 4t \\
 300 &= 380 - 4t \\
 300 - 380 &= 380 - 4t - 380 \\
 -80 &= -4t \\
 \frac{-80}{-4} &= \frac{-4t}{-4} ; \quad t = 20 \text{ weeks}
 \end{aligned}$$

Example 3: How much will Larry Large weigh after 38 weeks?

$$\begin{aligned}
 w(t) &= 380 - 4t \\
 w(38) &= 380 - 4 \cdot 38 \\
 w(38) &= 380 - 152 \\
 w(38) &= 228 \text{ lbs}
 \end{aligned}$$

Example 4: Sally Sadsack is leaving home to see the world. Her old junk VW will only do 45 mph. Specify the starting quantity, and the rate. Then write her distance from home (in miles) as a function of time (in hours), $d(t)$.

t, hrs	d(t), miles
0	$0 + 45(0) = 0$
1	$0 + 45(1) = 45$
2	$0 + 45(2) = 90$
3	$0 + 45(3) = 135$

starting quantity = 0 miles
rate = 45 mi/hr

$$d(t) = 0 + 45t$$

$$d(t) = 45t$$

Example 5: What is Sally's distance from home after 8 hrs?

$$d(t) = 45t$$

$$d(8) = 45 \cdot 8$$

$$d(8) = \boxed{360 \text{ miles}}$$

Example 6: After how many hours will Sally be exactly 287 miles from home?

$$d(t) = 45t$$

$$287 = 45t$$

$$\frac{287}{45} = \frac{45t}{45} ; t = \boxed{6.3\bar{7} \text{ hrs}}$$

In both examples above the rate was with respect to **time**. For example, lbs per **week** and miles per **hour**.

Notice that when we say **lb per week**, we can also write it as **lb/week**.
 Notice when we say **miles per hour**, we can also write it as **miles/hour**.

Notice that whatever follows the word “**per**” goes on the **bottom**.

Anything written in the form (something)/(something else) is specifying a **rate**.

Rates are very often with respect to time ...some quantity per some unit of time. However, as the next example shows, rates do not necessarily have to be with respect to time.

Example 7: A certain football team averages 4.5 yards/play. They start a drive on their own 20 yard line. Write a function that tells how far d from their own goal line they will be as a function of plays p . Specify a starting quantity and the rate.

p , plays	$d(p)$, yards
0	$20 + 4.5(0) = 20$
1	$20 + 4.5(1) = 24.5$
2	$20 + 4.5(2) = 29$
3	$20 + 4.5(3) = 33.5$

starting quantity = 20 yd
rate = 4.5 yd/play

$$d(p) = 20 + 4.5p$$

Example 8: How far will the team be from their own goal line after 7 plays?

$$\begin{aligned} d(p) &= 20 + 4.5p \\ d(7) &= 20 + 4.5(7) \\ d(7) &= 20 + 31.5 = \boxed{51.5 \text{ yd.}} \end{aligned}$$

Example 9: How many plays will it take for the team to be 65 yards from their own goal line?

$$\begin{aligned} d(p) &= 20 + 4.5p \\ 65 &= 20 + 4.5p \\ 65 - 20 &= 20 + 4.5p - 20 \\ 45 &= 4.5p \\ \frac{45}{4.5} &= \frac{4.5p}{4.5} ; \quad \boxed{p = 10 \text{ plays}} \end{aligned}$$

Assignment:

1. A water tank initially has 120 gallons of water in it. A pipe delivers water into the tank at the rate of 18 gal/min. Specify the starting quantity, the rate, and the water in the tank as a function of time.

t, min	w(t), gal

2. Referring to problem 1, after how many minutes will there be 233 gal of water in the tank?

3. Referring to problem 1, how many gallons of water will be in the tank after 20 minutes?

4. The Cats have a tremendous team this year and score points at the rate of 52 points per game. A new sports writer joins a newspaper midway through the season after the Cats have already racked up 156 points. He wants to write a function to know how many points p to expect them to have scored after g more games. Write this function along with a starting quantity and rate.

g, games	p(g), points

5. Referring to problem 4, how many more games will it take for the Cats to bring their total to 312 points?

6. Referring to problem 4, how many points will the Cats have scored by the end of 8 more games?

7. Miss Carol Spendmore is not very good at budgeting her money. She notices that her bank account is declining by \$44 dollars per day. If she initially had \$500 in the bank, write a function that describes her bank balance b as a function of time t in days. Specify a starting amount and rate,

t, days	b(t), dollars

8. How many days will it be until Miss Spendmore of problem 7 is flat broke?

9. At the rate Miss Spendmore of problem 7 is spending money, how much will her bank balance be at the end of 8 days?

10. Farmer Jones sells each apple for 32 cents. Write a function for the cost c for a box of a apples. Specify the starting quantity and the rate.

a, apples	c(a), dollars

11. How many apples does Farmer Jones of problem 10 need to sell in order to make \$6.08?

12. If Farmer Jones of problem 10 sells 302 apples, how much money will he make?

13. Express the ratio form of the rate 4 lb/day in “per” form.

14. Express the rate “ 18.3 counts per day” in ratio form.