Unit 7: Lesson 03 — Perpendicular and parallel lines

An important relationship between two different lines relates to the **angle between the two lines**. In this lesson we will examine the relationship between parallel and perpendicular lines.

Parallel lines:

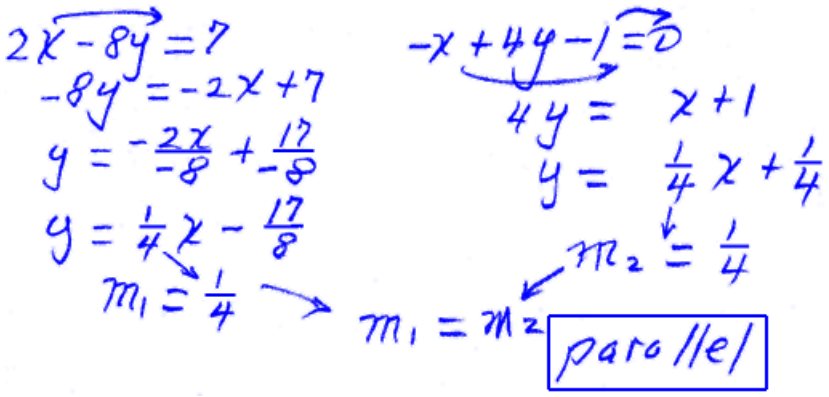
Slopes are equal. ($m_1 = m_2$)

Perpendicular lines:

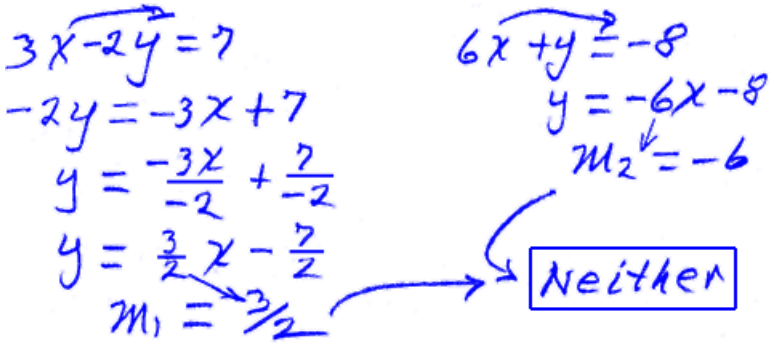
Slopes are negative reciprocals of each other. ($m_1 = -1/m_2$)

Notice that if $m_1 = -1/m_2$ is cross-multiplied, the result is **$m_1(m_2) = -1$**.

Example 1: Examine the two lines given by $2x - 8y = 7$ and $-x + 4y - 1 = 0$ to determine if they are parallel, perpendicular or neither.



Example 2: Examine the two lines given by $3x - 2y = 7$ and $6x + y = -8$ to determine if they are parallel, perpendicular or neither.



Example 3: Examine the two lines given by 4x – 12y = 2 and 6x +2y – 7 = 0 to determine if they are parallel, perpendicular or neither.

$4x - 12y = 2$
$-12y = -4x + 2$
$y = \frac{-4x}{-12} + \frac{2}{-12}$
$y = \frac{1}{3}x - \frac{1}{6}$
$m_1 = \frac{1}{3}$

$6x + 2y - 7 = 0$
$2y = -6x + 7$
$y = \frac{-6x}{2} + \frac{7}{2}$
$y = -3x + \frac{7}{2}$
$m_2 = -3$

$m_1 m_2 = \frac{1}{3}(-3) = -1 \rightarrow$ perpendicular

Example 4: Write the equation of a line that passes through (4, - 2) and is parallel to the line given by 4x + y = 11.

$4x + y = 11$
$y = -4x + 11$
$m_1 = -4$

$y = mx + b$
$y = -4x + b \quad (4, -2)$
$-2 = -4(4) + b$
$-2 + 16 = b$
$14 = b$

$y = mx + b$
$y = -4x + 14$

Example 5: Write the equation of a line that passes through (-1, 9) and is perpendicular to the line given by -x + 5y + 8 = 0.

$-x + 5y + 8 = 0$
$5y = x - 8$
$y = \frac{1}{5}x - \frac{8}{5}$
$m = \frac{1}{5}$
$m_\perp = -5$

$y = mx + b$
$y = -5x + b \quad (-1, 9)$
$9 = -5(-1) + b$
$9 = 5 + b$
$9 - 5 = b$
$4 = b$

$y = mx + b$
$y = -5x + 4$

See **Enrichment Topic C** for how to apply the equations of lines in two dimensions to the solutions of **two-dimensional inequalities**.

Assignment: In problems 1-5, determine if the two lines given by the pair of equations are parallel, perpendicular, or neither.

1. $x + y = 7$; $4x - 6 = 4y$

2. $3/4\ y = x - 6$; $x + 4/3y = 1$

3. $x = y$; $3 + 6y = 6x$

4. $y = 2x - 1$; $10x - 5y = 2$

5. $(1/5)x - y + 7 = 0$; $3y = -15x + 11$

6. Write the equation of a line that passes through (5, -2) and is parallel to the line given by $-3x + y = 10$.

7. Write the equation of a line that passes through (5, -2) and is perpendicular to the line given by $-x - y = 22$.

8. Write the equation of a line that has a y-intercept of 5 and is parallel to the line given by $-2x + 10y = 7$.

9. Write the equation of a line that has an x-intercept of -4 and is perpendicular to the line given by $x + 2y = 1$.

*10. Write the equation of a line that passes through (8, -2) and is parallel to the line connecting (4, 5) and (6, 15).

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11. If line #1 has a slope of m_1 and line #2 has a slope of m_2, what relationship must exist between lines #1 and #2 if $m_1(m_2) = -1$?