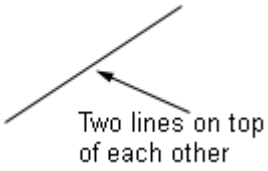
Unit 9: Lesson 01

The meaning of a solution to a system of linear equations

Consider the following two equations:

$2x - 4y = 9$
$11x - 5y = -8$

What does it mean to **solve this system of equations**? Very simply, it means to **find all the points of intersection** of these two lines. In general there are three distinct possibilities as shown below:

The two lines intersect in a single point. The *x* and *y* values of that point are the solutions to the system.	The two lines never intersect because the lines are parallel but separate.	The two lines are directly on top of each resulting in an infinite number of intersection points.
		

So, how can we tell by just looking at the equations of two lines which of the three pictures above represents their orientation?

If the **slopes of the two lines are different**, then it's the left picture above and we have only **one point** of intersection.

If the **slopes are the same** and the **y-intercepts are different**, then it's the middle picture above and we have **no points** of intersection.

If the **slopes are the same** and the **y-intercepts are the same** (they are actually the same line), then it's the right-hand picture above and there are **infinitely many points** of intersection.

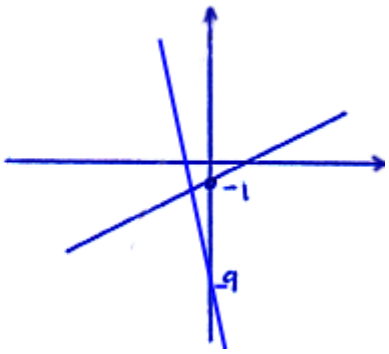
In each example below, examine the slope and y-intercept. Then tell how many points are in the solution set of the system. Make a rough sketch of the lines.

Example 1: $-4x = y + 9$ and $y = x - 1$

$-4x - 9 = y$ $\quad\downarrow$

$m_1 = -4$ $\quad m_2 = 1$

Slopes are different
(one intersection point)

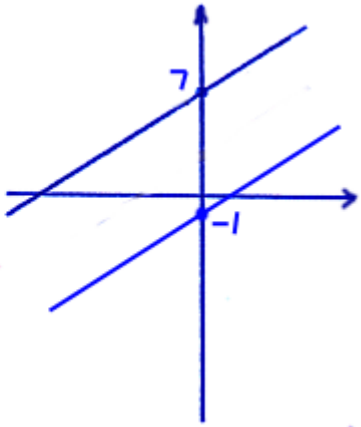


Example 2: $-x + y = 7$ and $y = x - 1$

$y = x + 7$ $\quad m_2 = 1$

$m_1 = 1$

$m_1 = m_2$ so the lines are parallel. They have dif. b values so separate.
(No Solution)

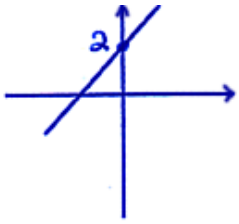


Example 3: $-x + y = 2$ and $6 + 3x - 3y = 0$

$y = x + 2$ $\quad -3y = -3x - 6$

$m_1 = 1$ $\quad y = x + 2$

$m_2 = 1$



Slope are the same; infact, the equations are the same. They sit on top of each other. So there are an infinite number of points as given by $y = x + 2$

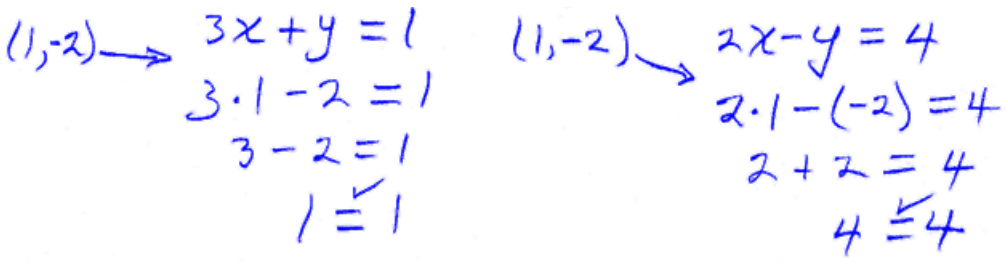
Infinitely many solutions

A point (x, y) can be proven to be a solution to a system of two linear equations

if it "satisfies" **both** equations. (When the point is substituted into both equations, they are **both** still true.)

Example 4: Prove that (x, y) = (1, -2) is a solution to this system:

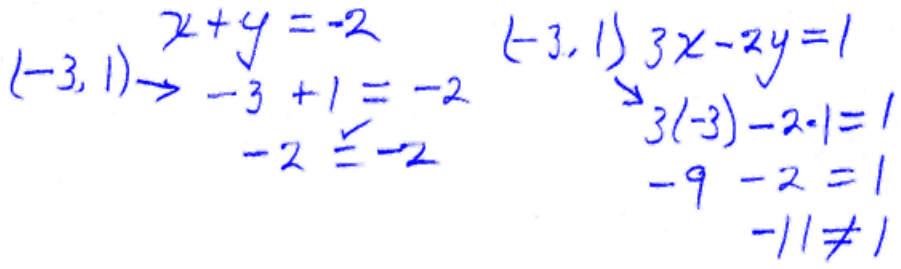
$3x + y = 1$
$2x - y = 4$



Yes, it is a solution since it satisfies both equations.

Example 5: Determine if (-3, 1) is a solution to this system:

$x + y = -2$
$3x - 2y = 1$



No, it is not a solution since it does not satisfy both equations.

Assignment: In the following problems, examine the slopes and y-intercepts of the two lines and then tell how many points are in the solution set of the system. Make a rough sketch of the lines.

1. $x + 4 = y \ \ and \ \ y = 3x - 8$

---

2. $2x - 3y = -1$ and $8x - 12y = -4$

---

3. The line given by (1, –5) & (6, –10) and $3y = -3x + 11$

4. Line A has a slope of 3/5 and crosses the y-axis at –8. Line B has a slope of 3/5 and crosses the y-axis at 22.

---

5. Line A has a slope of 1 and crosses the y-axis at 3. Line B has a slope of 3 and crosses the y-axis at –1.

---

6. The product of the slope of the two lines is –1. Hint: This has something to do with the orientation of the line (parallel, perpendicular, etc).

---

7. $y = 4.1x + 2 \quad and \quad 2y = 4.1x + 2$

8. The line connecting (0, 0) & (1,1) and the line connecting (–2, –2) & (5, 5)

---

9. $y = 3$ and $x = 3$

---

10. $y = 4$ and $y = -5$

11. Determine if (x, y) = (2, -3) is a solution of this system:

$x - 2y = 8$
$2x + y = 1$

---

12. Determine if (x, y) = (3, 6) is a solution of this system:

$4x - 2y = 0$
$x - 3y = 1$