

Blue Pelican
Calculator Appendices



Version 1.01

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Graphing Calculator Appendices

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Calculator Appendix A

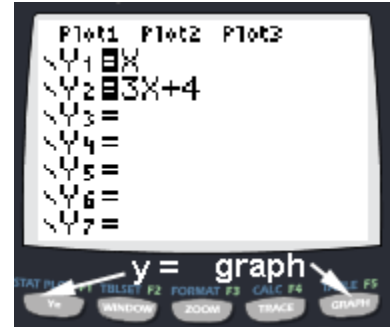


(Special marking of graphed functions)

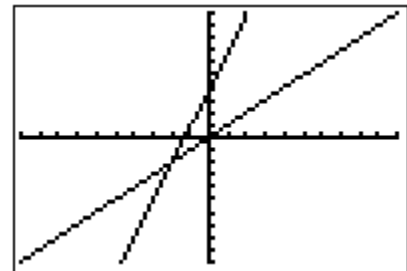
Multiple functions can be simultaneously graphed by pressing the **Y=** button. Enter the first function beside **Y1=** and others beside **Y2=**, etc.

For the sake of illustration, the functions $y = x$ and $y = 3x + 4$ are shown here.

After entering the functions, graph them by pressing the **GRAPH** button.



The two graphed functions will appear as shown here. (If not, press **ZOOM | 6: ZStandard** ... the zoom button is the middle button in the picture above.)



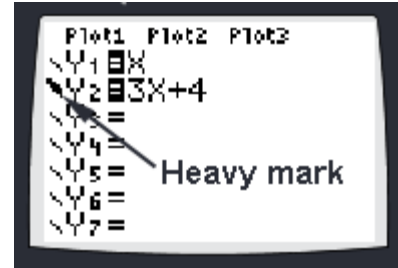
Although it is easy to tell which graph belongs to which function in this particular example, sometimes it is not this easy. This is especially true when there are several functions intertwined or when deep zooms allow only tiny portions to be viewed.

In the present example we will show how to **specialy mark one of the functions** so there will be **no doubt concerning its identity** on the graph screen.

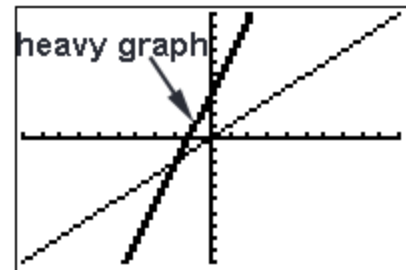
We will mark the steeper of the two lines ($Y2 = 3x + 4$) by moving with the **UP** and/or **DOWN ARROWS** until the cursor is on the **Y2** function.



At this point we want to move the cursor to the left of **Y2**. Do this by pressing the **LEFT ARROW** twice. Press **ENTER** and the icon to the left of **Y2** will turn into a heavy mark. (Repeated pressing of **ENTER** will toggle through other options.)



Now press the **GRAPH** button and the **Y2** function will be observed to be displayed with a heavier graph so as to associate it with the proper function.





Calculator Appendix B

(Graphing vertical lines)



Generally it is not possible to graph a perfectly vertical line on a TI 84 graphing calculator and have it behave as an ordinary function in the sense that it is possible to find its intersection with other functions.

However, it is possible to create a line so steep that when viewed at a particular zoom level, it essentially looks and behaves as a vertical line.

As an example, let's suppose we need the vertical line $x = 5$. This cannot directly be entered using the **Y=** button because that particular screen designates functions with **Y1**, **Y2**, etc. (not **X1**, **X2**, etc.)

For the particular zoom area you intend to use, pick a y value that is approximately in the middle of the zoom box (let's call it y_{mid}). Use the point-slope form for a line and with the point $(5, y_{mid})$ and a very large, negative slope of $m = 999999999999$. Thus, the point-slope equation of this line becomes:

$$Y1 - y_{mid} = -999999999999(x - 5)$$

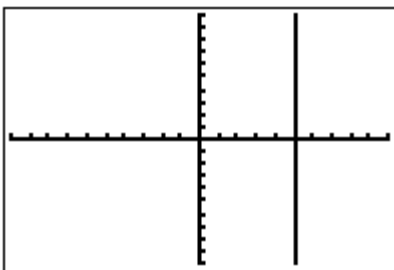
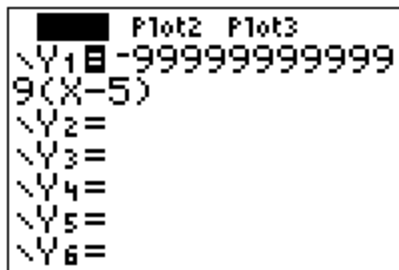
$$Y1 = -999999999999(x - 5) + y_{mid}$$

Most zooms are such that the x -axis is visible in the zoom area, so a safe value of y_{mid} to use is 0. In that case our equation would become:

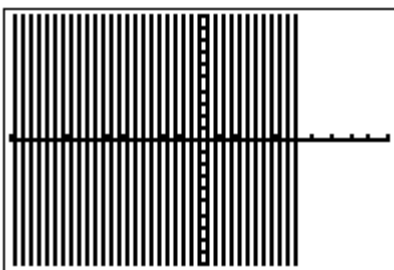
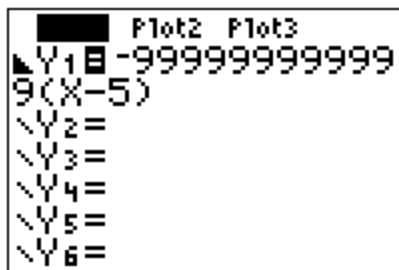
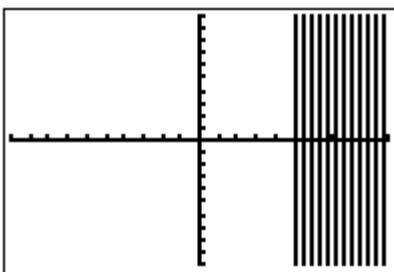
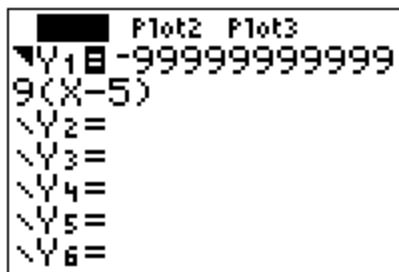
$$Y1 = -999999999999(x - 5) + 0$$

$$Y1 = -999999999999(x - 5)$$

For a zoom given by **ZOOM | 6: ZStandard** the display would appear as shown here.



Essentially the same result can be achieved with a positive slope of $m = 999999999999$; however, the **negative slope must be used** to obtain proper left/right shading when graphing the inequalities $x < 5$ or $x > 5$.



Notice the greater-than \blacktriangleright and less-than \blacktriangleleft icons just to the left of **Y1**. These are set by moving the cursor to the left of **Y1** with the **LEFT ARROW** key. Toggle through the various option by pressing **ENTER**.



Calculator Appendix C

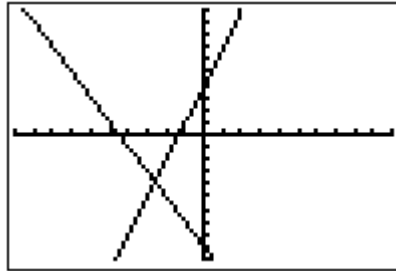
(Finding intersection points)



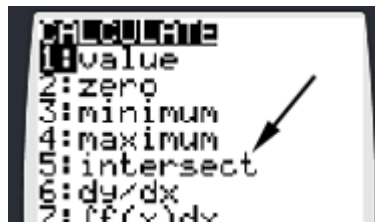
Consider finding the intersection point of two functions ($y = 3x + 4$ and $y = -2x - 9$). With **Y=**, enter the two functions and press **GRAPH**.

```

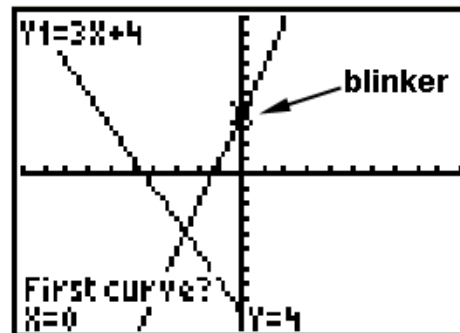
Plot2 Plot3
Y1=3X+4
Y2=-2X-9
Y3=
Y4=
Y5=
Y6=
Y7=
  
```



To initiate the process of finding the intersection, press **2nd | CALC**. From the resulting menu choose **5: intersect** by pressing the **DOWN ARROW** button thus moving the cursor all the way down to **5:** and then pressing **ENTER**, or by pressing the **5** button.

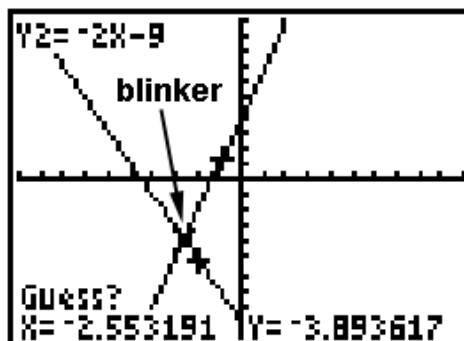
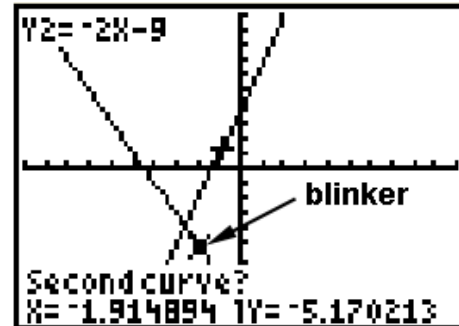


The next screen will show the graph along with a question, "First curve?". It is asking for either one of the curves. A blinker will be positioned on one of the "curves" (actually a line in this example) that is coincidentally also on the y-axis in this example.



The blinker can be moved along the present curve with the **LEFT/RIGHT ARROWS** (The **UP/DOWN ARROWS** jumps between curves). When satisfied that the blinker is distinctly on one of the curves, press **ENTER**.

At this point another question will appear asking for the second curve. Again, move the blinker with the **LEFT/RIGHT ARROWS** until satisfied that the blinker is distinctly on the other curve. Press **ENTER**.

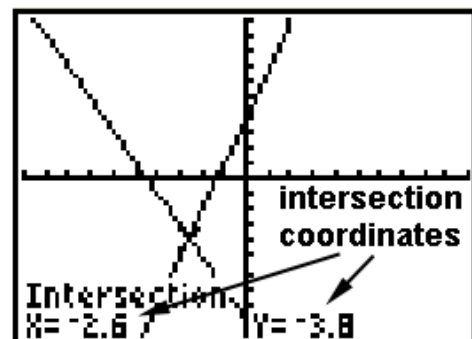


Finally, a third question is presented asking for a, "Guess?" Using the **LEFT/RIGHT ARROWS**, position the blinker approximately over the intersection point and press **ENTER**.

Students often ask why a guess is necessary. It's because the curves are often more complicated than our simple example and **intersect in more than one point**. The calculator needs to know which point we want, so the software inside the calculator insists on a "guess" in all cases.

The coordinates of the point of intersection are now shown at the bottom of the graph screen.

For simple cases like that depicted here in which there is only one intersection point, it is possible to get the answer with three quick **ENTER**'s after choosing **5: intersect**. (No need to adjust positions with the **LEFT/RIGHT ARROWS**.)





Calculator Appendix D



(Graphical solutions to absolute value problems)

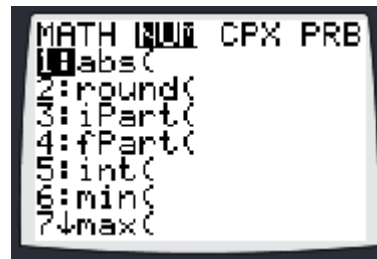
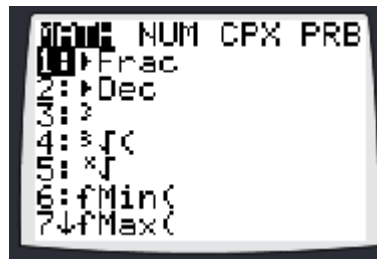
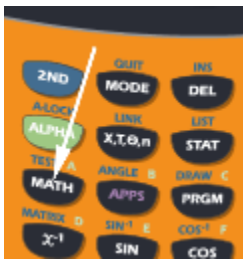
Consider the solution to the problem $|x - 4| = 5$. To solve this on a graphing calculator press **Y=** and enter:

$$Y_1 = \text{abs}(x-4)$$

$$Y_2 = 5$$

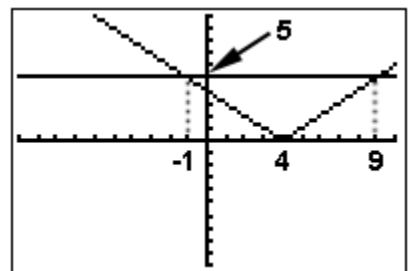
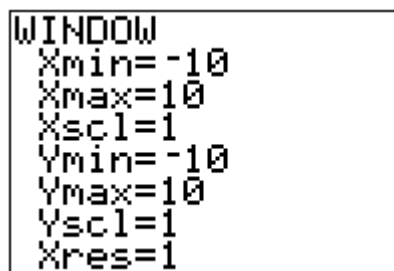
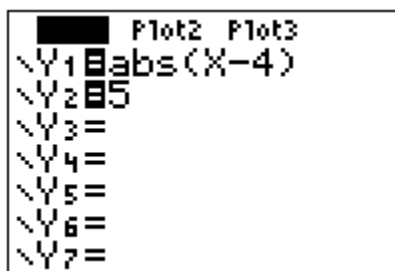
Access the absolute value function (given by **abs** above) with:

Math | Num | 1: abs(



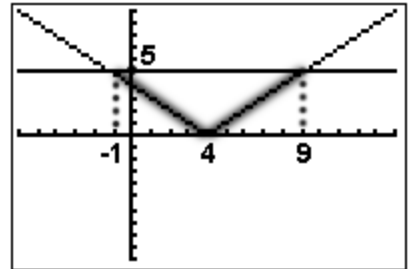
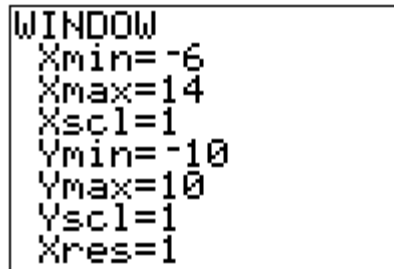
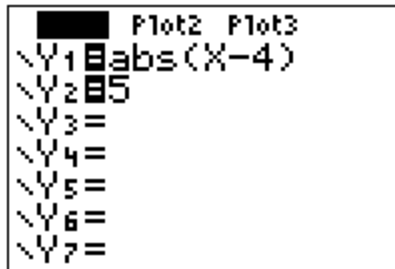
The x-coordinates of the two intersection points of these two functions are the solutions to this problem. (See Appendix C for how to find intersection points.)

From the graph below, we see that the solutions are $x = -1$ and $x = 9$.



Now consider the same absolute value quantity, but this time as an inequality:

$$|x - 4| \leq 5$$

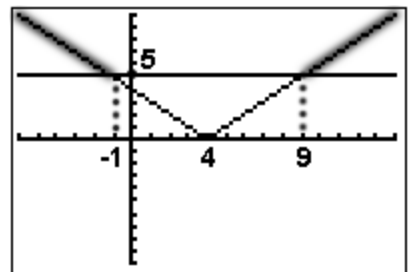
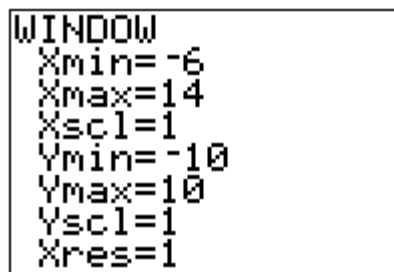
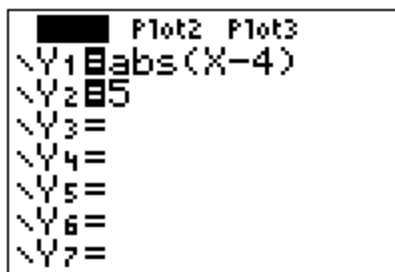


The “heavy” part of the graph above is the area in which the **Y1** function is less than (**below**) the **Y2** function. The x values that correspond to this constitute the solution:

$$-1 \leq x \leq 9 \quad \text{which is equivalent to } x \geq -1 \quad \text{and} \quad x \leq 9$$

Finally, consider the inequality:

$$|x - 4| \geq 5$$



The “heavy” part of the graph above is the area in which the **Y1** function is less than (**above**) the **Y2** function. The x values that correspond to this constitute the solution:

$$x \leq -1 \quad \text{or} \quad x \geq 9$$



Calculator Appendix E

(Graphing inequalities in two variables)



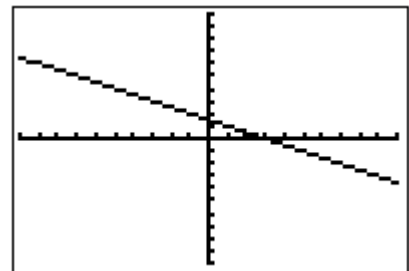
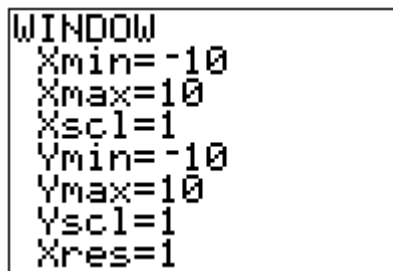
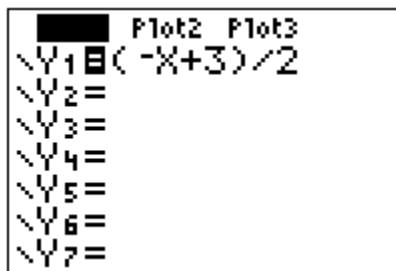
Consider the display of the following inequality on a graphing calculator.


$$x + 2y \geq 3$$

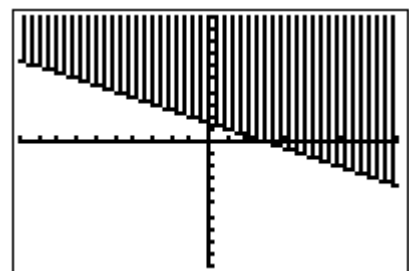
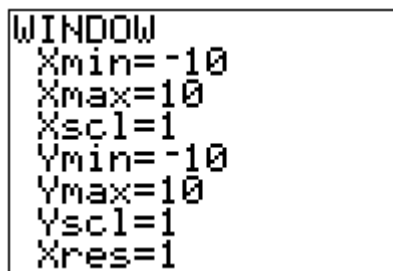
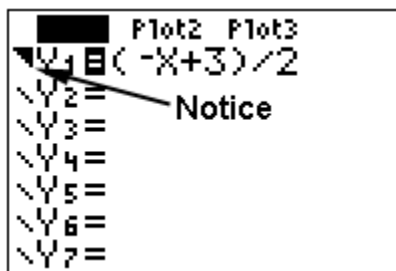
The first step is to solve for y :


$$y \geq (-x + 3)/2$$

Press the **Y=** button on the calculator and enter $(-x + 3)/2$ for **Y1**. Press **GRAPH** and observe that a **line** is graphed:




In order to turn this into a true inequality with a shaded region, use the **LEFT ARROW** to move the cursor to the left of **Y1 =**. Once the cursor is in that position, repeatedly press **ENTER** until this icon () appears. This causes the function (in this case a line) to be shaded **above**. Press **GRAPH** again and shading will be observed **above** the line.



To graph the inequality $y \leq (-x + 3)/2$ which will require shading **below** the line, use the **LEFT ARROW** to move the cursor to the left of **Y1 =**. Once the cursor is in that position, repeatedly press **ENTER** until this icon () appears. Press **GRAPH** and shading will be observed **below** the line.

```

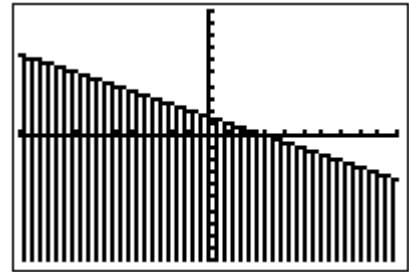
Plot2 Plot3
Y1  (-X+3)/2
Y2 =
Y3 =
Y4 =
Y5 =
Y6 =
Y7 =



```

```

WINDOW
Xmin=-10
Xmax=10
Xscl=1
Ymin=-10
Ymax=10
Yscl=1
Xres=1

```



Similarly, the icons  and  will result in **any function** (not just lines) being shaded respectively above or below the graph.

Think of it this way:

With , **Y1= ...** effectively becomes **Y1 ≥ ...**, and

With , **Y1= ...** effectively becomes **Y1 ≤ ...**



Calculator Appendix F



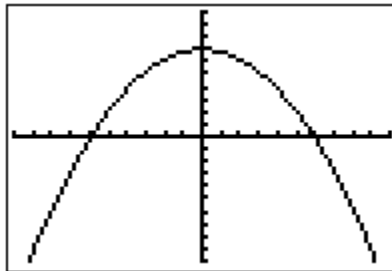
(Restricting the domain of a function)

It is possible to restrict the domain of a graphed function on a graphing calculator so as to make only a portion of the graph visible.

We will consider the function $f(x) = -.2x^2 + 7$. Press **Y=**, enter the function as **Y1**, and then press **GRAPH**.

```

Plot2 Plot3
Y1 = -.2X^2+7
Y2 =
Y3 =
Y4 =
Y5 =
Y6 =
Y7 =
  
```



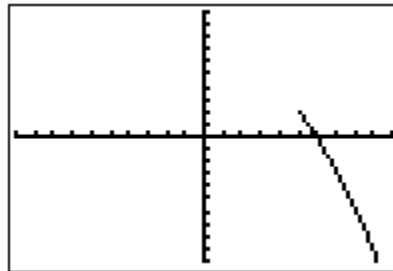
```

WINDOW
Xmin=-10
Xmax=10
Xscl=1
Ymin=-10
Ymax=10
Yscl=1
Xres=1
  
```

To restrict the domain to $x > 5$, append $/(x > 5)$ to the end of the function specified by **Y1**. Be sure to enclose the function in parenthesis.

```

Plot2 Plot3
Y1 = (-.2X^2+7)/(X
Y2 =
Y3 =
Y4 =
Y5 =
Y6 =
  
```



The greater than symbol ($>$) given in **Y1** above is accessed with **2nd | MATH(Test) | 3: >** as shown here.

```

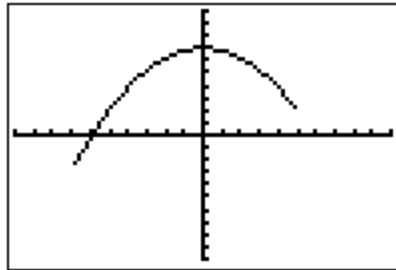
2ND LOGIC
1: =
2: +
3: >
4: <
5: <=
6: >=
7: <>
  
```

Consider restricting the same function to the domain $-7 < x < 5$ by appending $/(x > -7 \text{ and } x < 5)$ to the end of the function specified by **Y1**.

```

P1ot2 P1ot3
\Y1 (-.2X^2+7)/(X
>-7 and X<5)
\Y2 =
\Y3 =
\Y4 =
\Y5 =
\Y6 =

```



The logic specifier **and** given in **Y1** above is accessed with **2nd | MATH(Test) | Logic | 1: and** as shown here.

```

TEST 1:and
2:or
3:xor
4:not(

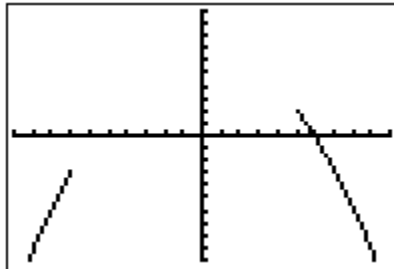
```

Consider restricting the same function to the domain $x < -7$ or $x > 5$ by appending $/(x < -7 \text{ or } x > 5)$ to the end of the function specified by **Y1**.

```

P1ot2 P1ot3
\Y1 (-.2X^2+7)/(X
<-7 or X>5)
\Y2 =
\Y3 =
\Y4 =
\Y5 =
\Y6 =

```



The logic specifier **or** given in **Y1** above is accessed with **2nd | MATH(Test) | Logic | 2: or**.



Calculator Appendix G



(Even and odd functions)

Consider the test to determine if a function is even: $f(x) = f(-x)$

This means if we graph $f(x)$ it should be identical to $f(-x)$.

A classical example of an even function is a parabola, for example, $f(x) = x^2 + 3$. If we graph $f(-x) = (-x)^2 + 3$, it should fall directly on top of $f(x)$. For confirmation of this, enter both functions with **Y=** on a graphing calculator. The results are shown below:

```

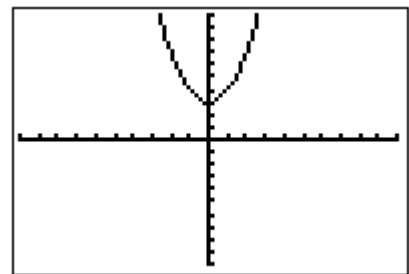
Plot2 Plot3
Y1=X^2+3
Y2=(-X)^2+3
Y3=
Y4=
Y5=
Y6=
Y7=

```

```

WINDOW
Xmin=-10
Xmax=10
Xscl=1
Ymin=-10
Ymax=10
Yscl=1
Xres=1

```



Consider the test to determine if a function is even: $f(x) = -f(-x)$

This means if we graph $f(x)$ it should be identical to $-f(-x)$.

A classical example of an even function is a cubic function, for example, $f(x) = .2x^3$. If we graph $f(-x) = -(.2(-x)^3)$, it should fall directly on top of $f(x)$. For confirmation of this, enter both functions with **Y=** on a graphing calculator. The results are shown below:

```

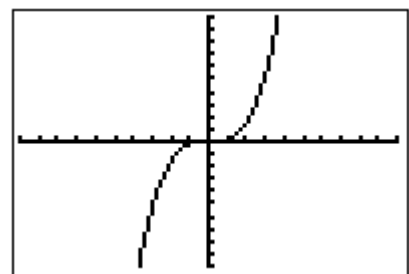
Plot2 Plot3
Y1=.2X^3
Y2=-(.2(-X)^3)
Y3=
Y4=
Y5=
Y6=
Y7=

```

```

WINDOW
Xmin=-10
Xmax=10
Xscl=1
Ymin=-10
Ymax=10
Yscl=1
Xres=1

```





Calculator Appendix H



(Inverse functions)

The functions $f(x) = 3x + 4$ and $g(x) = (x - 4)/3$ are **inverse functions** of each other. This is confirmed when the two functions are simultaneously graphed. They are reflections of each other across a 45° line ($y = x$). All three are graphed below after entering the functions with **Y=**:

```

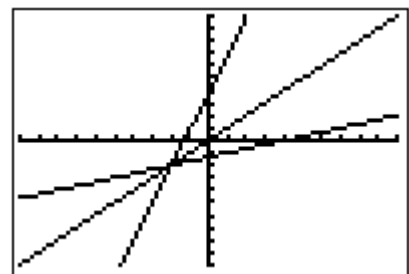
Plot2 Plot3
\Y1=3X+4
\Y2=(X-4)/3
\Y3=X
\Y4=
\Y5=
\Y6=
\Y7=

```

```

WINDOW
Xmin=-10
Xmax=10
Xscl=1
Ymin=-10
Ymax=10
Yscl=1
Xres=1

```



The above test for reflection across the line $y = x$ is a **graphical test**. It would seem that on a graphing calculator that it would not be possible to do the **algebraic test** of $f(g(x)) = x$; however, it is possible if we enter

$$Y1 = 3((x-4)/3) + 4 \quad \text{which is } f(g(x))$$

If this graphs as a line through the origin with a slope of 1, then it is the line given by $y = x$. This is, in fact, the case as shown below, proving that they are inverses. (Actually, it would also be necessary to show that $g(f(x))$ also yields the line $y = x$.)

```

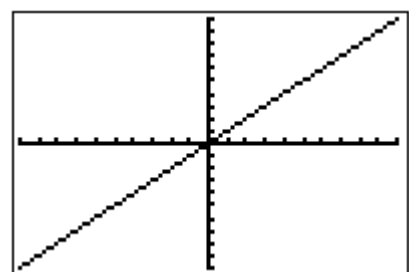
Plot2 Plot3
\Y1=3((X-4)/3)+4
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=

```

```

WINDOW
Xmin=-10
Xmax=10
Xscl=1
Ymin=-10
Ymax=10
Yscl=1
Xres=1

```





Calculator Appendix I



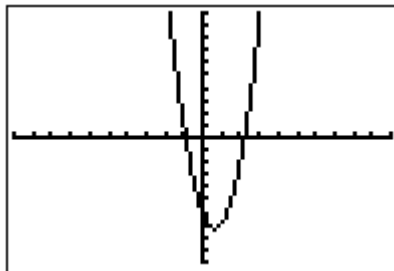
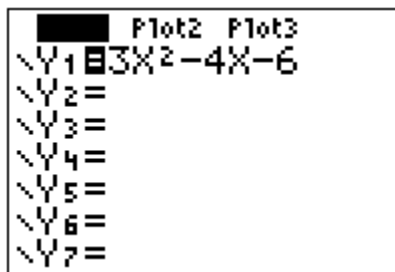
(Finding roots)

The root of a function is simply the x value of where it crosses the x -axis. It is possible for a function to have several roots.

Other names for **root** are **zero** and **x -intercept**. On a graphing calculator it is called a zero to emphasize that the value of the function is $y = 0$.

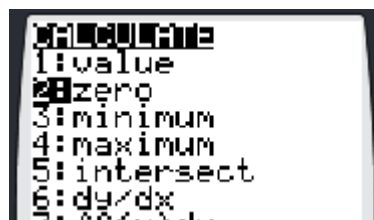
Consider the function $y = 3x^2 - 4x - 6$. Finding the roots of this function is equivalent to solving the equation $3x^2 - 4x - 6 = 0$.

Begin by pressing the **Y=** button and entering the function as **Y1**. Then press **GRAPH**.

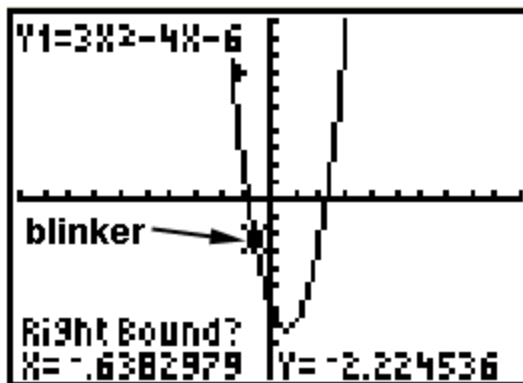
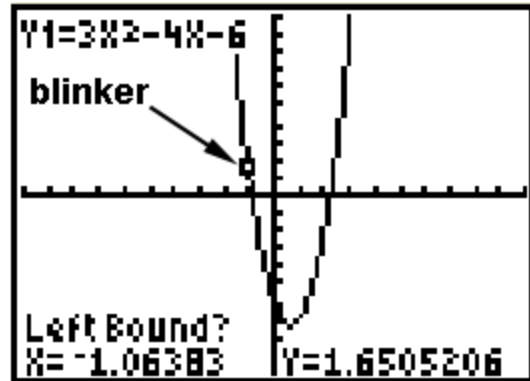


Clearly, from the graph there are two roots. We will concentrate on the left root.

To initiate the process of finding a root, press **2nd | CALC**. From the resulting menu choose **2: zero** by pressing the **DOWN ARROW** button thus moving the cursor down to **2:** and then pressing **ENTER**, or by just pressing the **2** button.

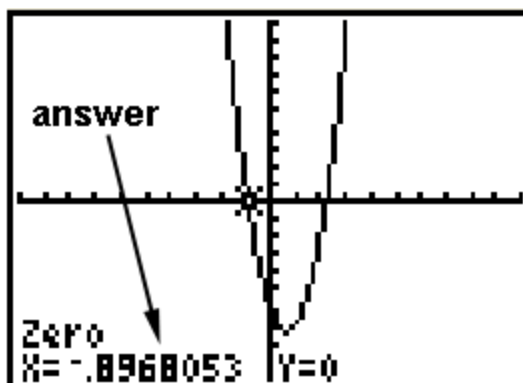
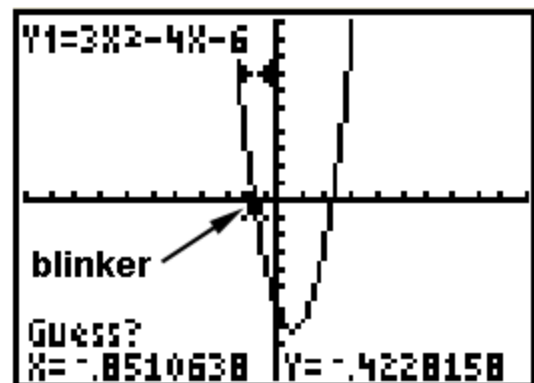


The next screen will show the graph along with a question, "Left Bound?". Move the blinker with the **LEFT/RIGHT ARROWS** until it is clearly to the left of the desired root (in this case, the left root). Press **ENTER** when satisfied with the position.



At this point another question will appear asking for the "Right Bound?". Move the blinker with the **LEFT/RIGHT ARROWS** until it is clearly to the right of the desired root. Press **ENTER**.

Finally, a third question is presented asking for a, "Guess?" Using the **LEFT/RIGHT ARROWS**, position the blinker approximately over the root and press **ENTER**.



At this point the answer is presented at the bottom of the screen.

The other root is found in a similar fashion.



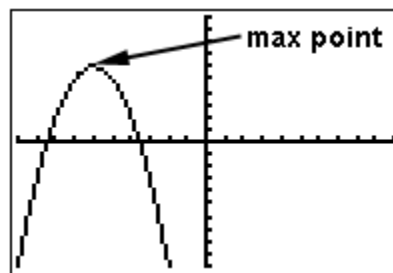
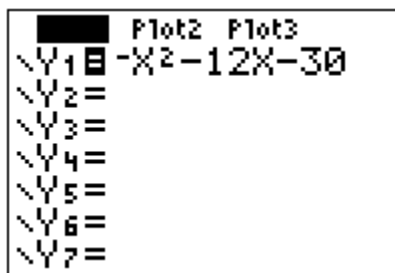
Calculator Appendix J



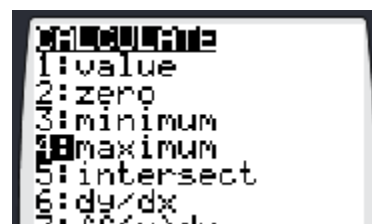
(Finding maximum and minimum points)

A **maximum point** of a function is a local point which has a greater value than all other points in its neighborhood. A function can have several maximums. A minimum point is similarly defined.

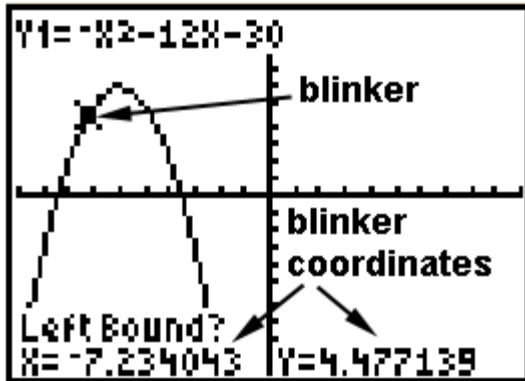
Consider the function $y = -x^2 - 12x - 30$. Our task is to find its single maximum. Obviously it's an "upside down" parabola and has a maximum point at the vertex of the parabola. Begin by pressing the **Y=** button and entering the function as **Y1**. Then press **GRAPH**.



To initiate the process of finding the maximum point, press **2nd | CALC**. From the resulting menu choose **4: maximum** by pressing the **DOWN ARROW** button thus moving the cursor down to **4:** and then pressing **ENTER**, or by just pressing the **4** button.

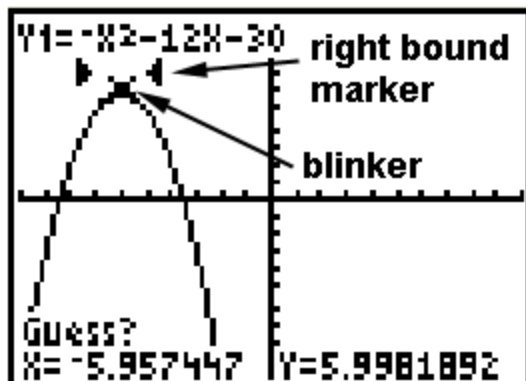
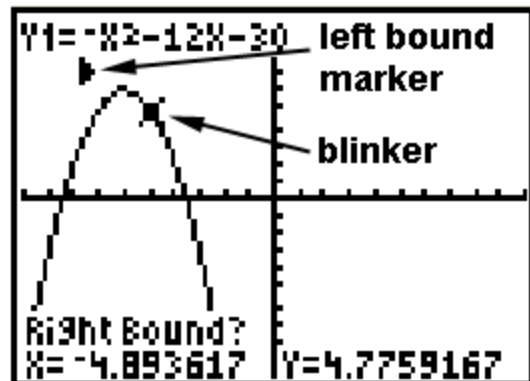


The instructions below will require moving the blinker with the **LEFT/RIGHT ARROWS**. Sometimes the blinker is not visible and it's not clear which way to move. Use the coordinates of the blinker at the bottom of the graph screen along with the zoom box coordinates to decide.



The next screen will show the graph along with a question, “Left Bound?”. Move the blinker with the **LEFT/RIGHT ARROWS** until it is clearly to the left of the desired maximum point. Press **ENTER** when satisfied with the position.

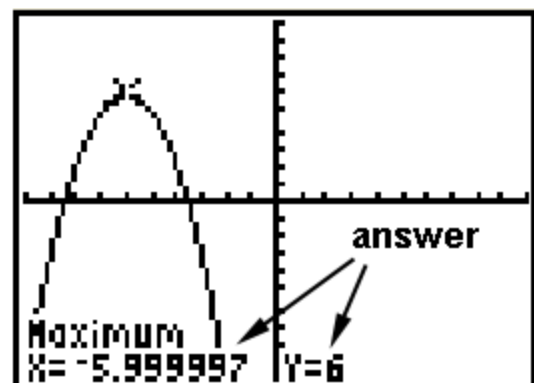
At this point another question will appear asking for the “Right Bound?”. Move the blinker with the **LEFT/RIGHT ARROWS** until it is clearly to the right of the desired maximum point. Press **ENTER**.



Finally, a third question is presented asking for a, “Guess?”. Using the **LEFT/RIGHT ARROWS**, position the blinker approximately over the maximum point and press **ENTER**.

The coordinates of the maximum point are now presented at the bottom of the screen.

Minimum points are found in a similar fashion.





Calculator Appendix K



(Calculations with the natural number e)

Just as π is a natural number, so is e . The value of e , an irrational number, is approximately 2.718...

Typically, what we do with e is raise it to a power. Access this with **2nd | LN**. This places $e^{\text{ ($ on the screen which means e raised to the power in the parenthesis. Enter the argument, close the parenthesis, and press **ENTER** for the result.



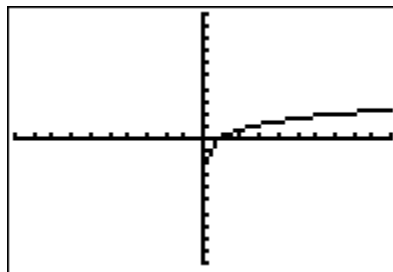
To see e to 9 decimal places, enter simply 1 as the argument. This gives e^1 which, of course, is just e . Also shown here is an evaluation of $e^{4.6603}$.

```
e^(1)
2.718281828
e^(4.6603)
105.6677777
```

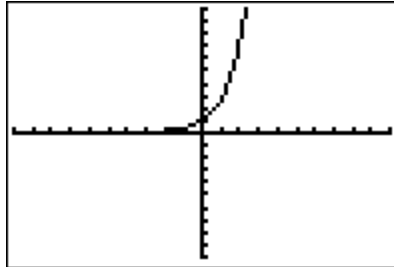
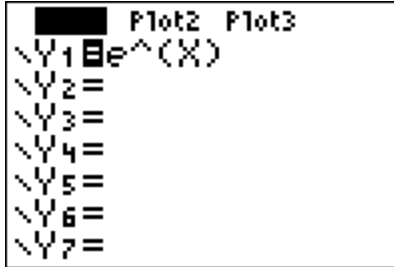
The reason e is associated with the LN button is that, that button gives the logarithm, base e . See Appendix L for a discussion of calculations of logarithms.

Press **Y=** and enter $\ln(x)$ for **Y1**. The graph of $y = \ln(x)$ is shown below. Notice that the curve **asymptotically approaches the y-axis** and has a **root of 1**.

```
Plot2 Plot3
\Y1=ln(X)
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```



Press **Y=** and enter e^x for **Y1**. The graph of $y = e^x$ is shown below. Notice that the curve **asymptotically approaches the x-axis** and has a **y-intercept of 1**.





Calculator Appendix L

(Logarithms)



Two different choices are available when computing logarithms; base 10, and base e :

- Base 10 (common logarithms, use the **LOG** button)
- Base e (natural logarithms, use the **LN** button)



Calculate the common log of 100. This can be written as $\log_{10}100$ but is normally written as $\log 100$. (When the base is omitted it's understood to be 10.)

The answer is 2 since this is the power to which 10 should be raised in order to equal 100. Shown here is how this problem is entered into the calculator. Notice that after pressing the **LOG** button, **log(** is displayed on the screen. Finish by entering 100 and a closing parenthesis. Press **ENTER** to get the answer (It's not necessary to close the parenthesis before pressing **ENTER**, but is highly recommended.)

log(100)

2

Similarly, calculate the natural log of 128.307. This can be written as $\log_e 128.307$ but is normally written as $\ln 128.307$ or $\ln(128.307)$.

This problem is asking the question, "To what power must e be raised in order to get 128.307. (e is the irrational, natural number 2.718...)

Press the **LN** button which results in the display **ln(** . Finish entering the argument, 128.307, and press **ENTER**.
The displayed results are shown here.

```
ln(128.307)
4.85442583
```

There are occasions in which a logarithm to a different base must be found. Consider the problem $\log_{13}(153.6692)$. Use the change-of-base formula as follows:

$$\log_{13}(153.6692) = \frac{\log(153.6692)}{\log(13)}, \text{ or}$$

$$\log_{13}(153.6692) = \frac{\ln(153.6692)}{\ln(13)}$$

According to the formulas above, a logarithm with **any base** can be expressed in **terms of** either common or natural logs.

For common logs the display on the calculator would be as follows:

```
log(153.6692)/log
(13)
1.962924619
█
```



Calculator Appendix M

(Scatter Plots)

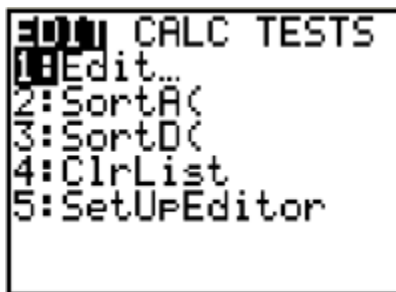


Consider the data in this table. These points can be displayed as a “scatter-plot” with the steps described below.

x	y
-9	-2.1
-5.2	-1
-2	2.04
2.3	5
6	6.11
8.9	8.2



The first step is to press the STAT button which yields the following display.



At this point either press **ENTER** or **1** indicating we wish to choose menu item 1 (editing/entering the list of x and y values).

Enter the x and y values choosing each position with the **Left/Right & UP/DOWN ARROWS**.

Enter the x values under list **L1** and the y values under list **L2**.

If **L1** and **L2** are not visible use the **LEFT/RIGHT ARROWS** so they display. If old lists are present use the **UP ARROW** to position the cursor on top of, for example **L1**, press **CLEAR**, and press **ENTER**.

L1	L2	L3	2
-9	-2.1	-----	
-5.2	-1		
-2	2.04		
2.3	5		
6	6.11		
8.9	8.2		
-----	-----		
L2(1) = -2.1			

The next step will be to press **2nd | STAT PLOT | 1 | On** so as to enable a scatter-plot to be displayed. This sequence is shown below. In the picture on the far right, use the **LEFT/RIGHT ARROWS** to choose between **On** and **Off** (choose **On**).



```

STAT PLOTS
1:Plot1...On
  [left] L1  L2  [right]
2:Plot2...Off
  [left] L1  L2  [right]
3:Plot3...Off
  [left] L1  L2  [right]
4↓PlotsOff
  
```

```

Plot1 Plot2 Plot3
On Off
Type: [scatter] [line] [bar]
      [line] [line] [line]
Xlist:L1
Ylist:L2
Mark: [square] + .
  
```

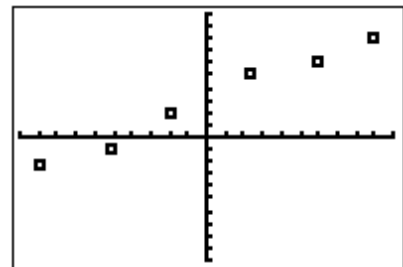
Finally, display the scatter plot by pressing the **GRAPH** button.

L1	L2	L3	Z
-9	-2.1	-----	
-5.2	-1		
-2.7	2.04		
2.3	5		
6	6.11		
8.9	8.2		

L2(1) = -2.1			

```

WINDOW
Xmin=-10
Xmax=10
Xscl=1
Ymin=-10
Ymax=10
Yscl=1
Xres=1
  
```



See Appendix N for how to produce the equation of a line that “best fits” this scatter-plot.



Calculator Appendix N

(Regression)



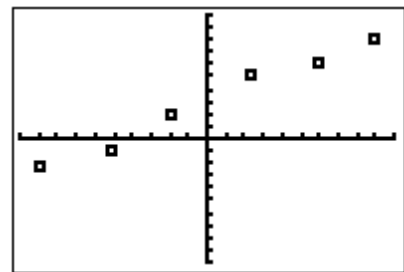
The process of regression is finding a function that “best-fits” a scatter-plot. In this Appendix a **linear regression** that determines the best-fit of a line will be demonstrated. Shown below is the data and the resulting scatter-plot from Appendix M that will be used here. That Appendix shows how to enter the data and produce the scatter-plot.

L1	L2	L3	Z
-9	-2.1	-----	
-5.2	-1		
-2.7	2.04		
2.3	5		
6.9	6.11		
	8.2		

L2(1) = -2.1			

```

WINDOW
Xmin=-10
Xmax=10
Xscl=1
Ymin=-10
Ymax=10
Yscl=1
Xres=1
  
```



With the above data already in the lists **L1** (the x values) and **L2** (the y values), press the **STAT** button. With the **RIGHT ARROW** move to **CALC** and then with the **DOWN ARROW** down to **4: LineReg(ax+b)**.

Press **ENTER** twice to produce the a and b values defining the best-fit line:

```

EDIT [2ND] TESTS
1:1-Var Stats
2:2-Var Stats
3:Med-Med
4:LinReg(ax+b)
5:QuadReg
6:CubicReg
7:QuartReg
  
```


```

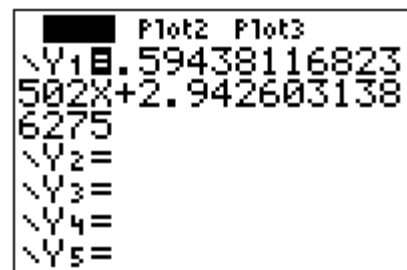
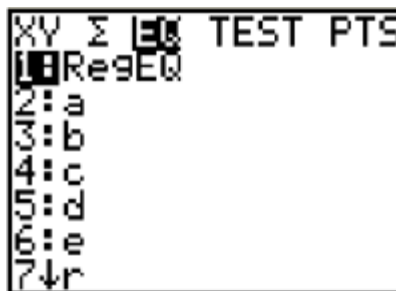
LinReg
y=ax+b
a=.5943811682
b=2.942603139
  
```

At this point we could press **Y=** and enter:

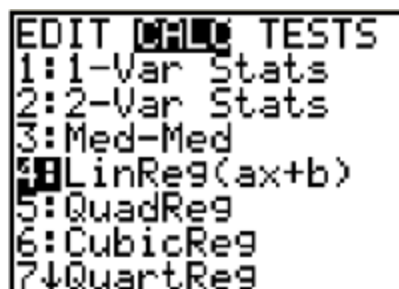
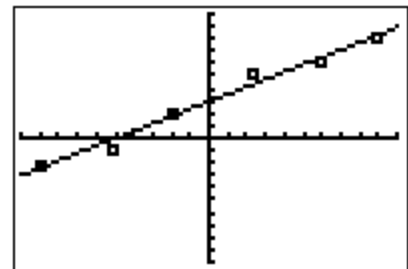
$$Y1 = .5943811682X + 2.942603139$$

However, there is an easier way. Rather than having to manually enter the a and b values for **Y1**, it is possible to automatically have them entered.

With the cursor on **Y1=** (on the **Y=** screen), press the **VAR** button,  choose **5: Statistics**, **RIGHT ARROW** across to **EQ**, and then down to **1: RegEQ**. At this point the regression equation for the best-fit is automatically entered for **Y1**.



Press **GRAPH** and if **STAT PLOT** is still enabled (see Appendix M), the following should display as the best-fit for for the scatter-plot.



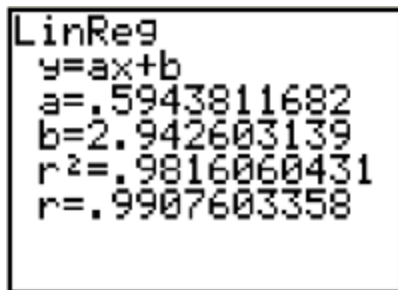
Some data does not lend itself to being best-fit with a line. For data that obviously “curves”, other type regressions are available (parabolic (QuadReg), cubic, exponential, etc.)

When doing various regressions on a set of data points, it is possible to get “scores” on the quality of each fit.



First, it will be necessary to turn on “diagnostics” so that a score can be displayed. Press **2ND** |

CATALOG, **Down ARROW** down to **1: Diagnostics ON**, and press **ENTER** twice to turn on diagnostics.



With diagnostics on, a regression will now display r^2 = “some number less than or equal to one”. Shown here is the same linear regression as above except now r^2 = .9816060431 is displayed along with a corresponding r value. r^2 is the square of the **correlation factor**, r , which can range from -1 to 1.

To eliminate the sometimes confusing negative r values, r^2 is often given as a figure of merit, were of, of course, $r^2 = 1$ is a **perfect score** as a result of a perfect fit.

For a linear regression, a negative r value indicates that the line of best-fit has a negative slope and the **correlation is said to be negative**. Likewise, a best-fit line with a positive slope is represented with a positive value of r and the **correlation is said to be positive**.



Calculator Appendix O



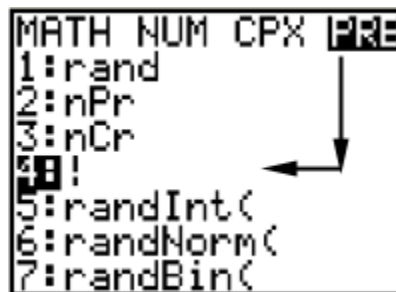
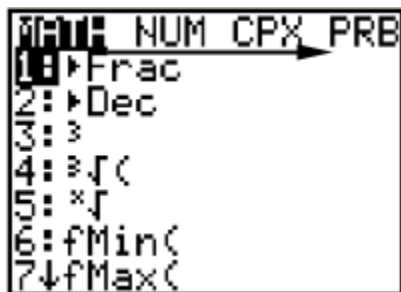
(Factorials, permutations, and combinations)

The factorial function is accessed via the **Math** button. For example, “four factorial” (written as 4!) evaluates as follows:

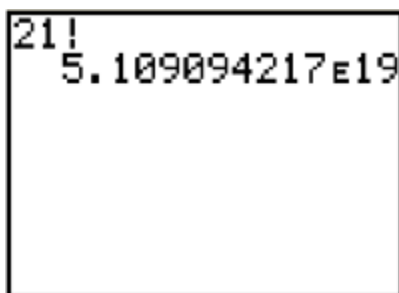
$$4! = 4(3)(2)(1) = 24.$$

Use the following sequence to evaluate, for example, 21! :

Enter 21, press the **Math** button, **RIGHT ARROW** across to **PRB**, **DOWN ARROW** to **4: !**, and press **ENTER**.



At this point you should have “21!” on the screen. Press **ENTER** one more time to evaluate this factorial:

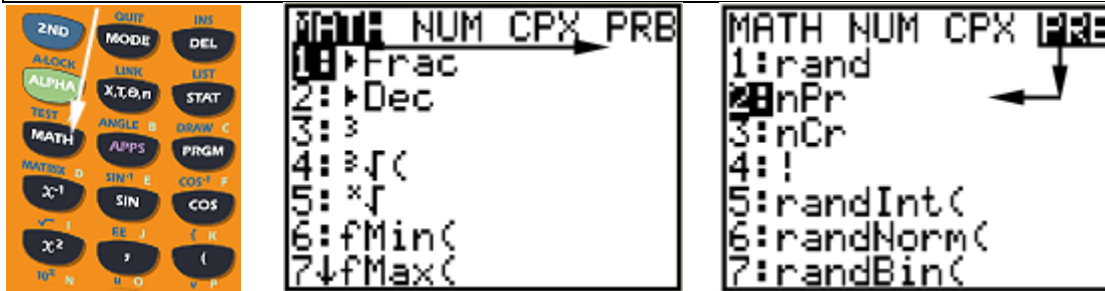


The permutations function is also accessed via the **Math** button. For example, nPr is read, “the number of permutations of n things taken r at a time”. The formal definition is:

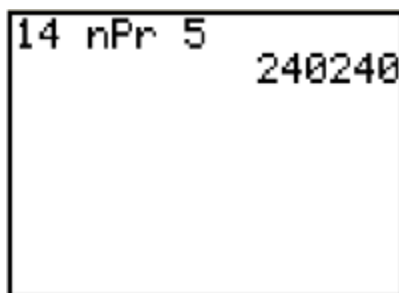
$$nPr = \frac{n!}{(n-r)!}$$

Use the following sequence to evaluate, for example, $14P_5$:

Enter 14, press the **Math** button, **RIGHT ARROW** across to **PRB**, **DOWN ARROW** to **2: nPr**, and press **ENTER**.



At this point you should have “14 nPr” on the screen. Enter 5 and the screen should now read “14 nPr 5”. Press **ENTER** one more time to evaluate the number of permutations:

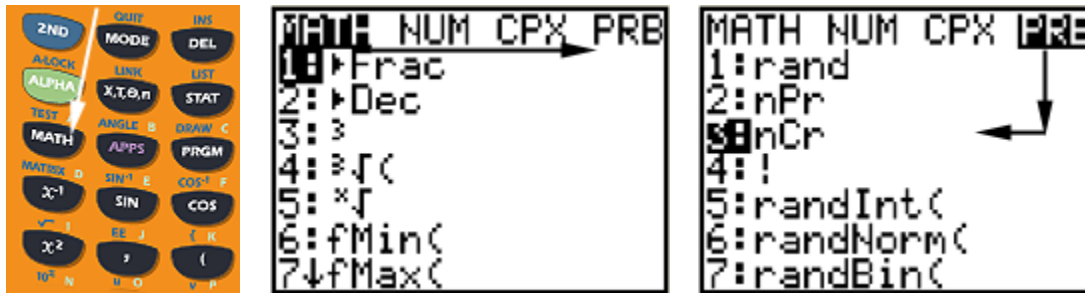


The combinations function is also accessed via the **Math** button. For example, nCr is read, “the number of combinations of n things taken r at a time”. The formal definition is:

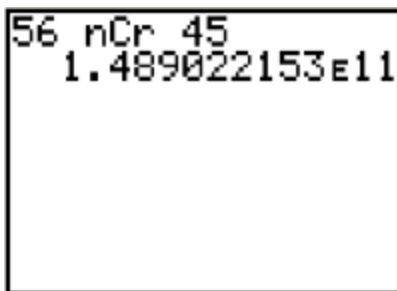
$$nCr = \frac{n!}{(n-r)!r!}$$

Use the following sequence to evaluate, for example, $56C45$:

Enter 14, press the **Math** button, **RIGHT ARROW** across to **PRB**, **DOWN ARROW** to **3: nCr**, and press **ENTER**.



At this point you should have “56 nCr” on the screen. Enter 45 and the screen should now read “56 nPr 45”. Press **ENTER** one more time to evaluate the number of combinations:





Calculator Appendix P

(Statistics)



Consider the following set of test grades:

{ 92, 99, 85, 95, 79, 64, 79, 91, 56, 82, 81 }

These numbers can be entered and the following statistics easily produced:

mean, median, standard deviation, lower-quartile, upper-quartile, min, max, ... and more.

Press the **STAT**  button and then select **EDIT** by pressing **ENTER**.

```

CALC TESTS
1:Edit...
2:SortA(
3:SortD(
4:ClrList
5:SetUpEditor
  
```

L1	L2	L3	Z
92			
99			
85			
95			
79			
64			
79			
L2(1)=			


Under **L1**, enter the first number in the list, press the **DOWN ARROW**, and then enter the next number. Continue until all numbers are entered. (If **L1** is not initially empty, **UP ARROW** on top of **L1**, press **CLEAR** and then **ENTER**.)

Now that the data is entered we are ready to display statistical facts about the data:

Press the **STAT**  button, **RIGHT ARROW** over to **CALC**.

Choose **1: 1-Var Stats**.
Press **ENTER** twice.

```

EDIT  TESTS
1:1-Var Stats
2:2-Var Stats
3:Med-Med
4:LinReg(ax+b)
5:QuadReg
6:CubicReg
7↓QuartReg
  
```

The statistics for the data in the **L1** list displays. Use the **DOWN-ARROW** to move down the display to view the remaining statistics.

```

1-Var Stats
x̄=82.09090909
Σx=903
Σx²=75795
Sx=12.91088336
σx=12.31004428
↓n=11

```

mean
standard dev

```

1-Var Stats
↑n=11
minX=56
Q1=79
Med=82
Q3=92
maxX=99

```

lower quartile
median
upper quartile

The following definitions apply to the some of the displayed statistics:

The **mean** (\bar{x}) is the average:

The **median** (Med) is the number in the middle of the **ordered** list:
(If there are two numbers in the middle, average them.)

The **lower quartile** (Q_1) is the median of the lower half of the numbers:

The **upper quartile** (Q_3) is the median of the upper half of the numbers:

(When there are an odd number of data points, the middle value is not used when finding either quartile.)

The **standard deviation** (σ) is a measure of the average “scatter” of the data.



Calculator Appendix Q



Matrix Fundamentals (elementary matrix math)

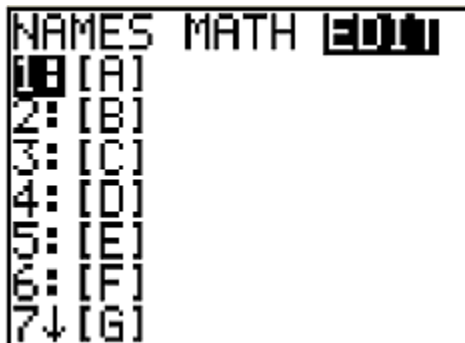
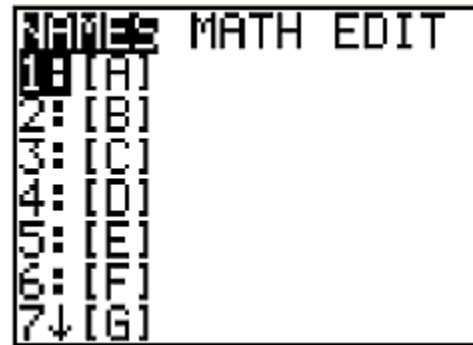
Consider the following matrix problem:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \quad C = \begin{bmatrix} -1 & -2 \\ -3 & -4 \end{bmatrix}$$

$$2A - 3(B - C)$$



The first step is to enter the matrices A , B , and C . Begin by pressing **2ND | MATRIX** resulting in this display. **RIGHT ARROW** twice across to **EDIT**.



Press **ENTER** and this screen appears. Use the **UP/DOWN ARROWS** to select the matrix we wish to edit (in this case **A**). Press **ENTER**.

This screen will appear asking for the dimensions of the **A** matrix in the upper right corner. Enter the number of rows first. Then press **ENTER**. Enter the number of columns and then **ENTER**.





MATRIX[A] 2 x2
 [1 2]
 [0 0]
 2, 1=3

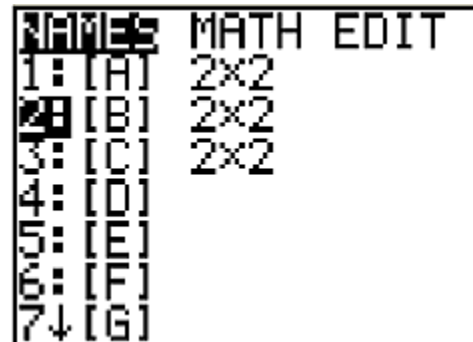
Enter each element of the **A** matrix pressing **ENTER** after each is entered. This display shows the next to last element (**3**) being entered.

Similarly, enter matrix **B** with the key sequence **2ND | MATRIX | EDIT | 2:[B] | ENTER**. In the same fashion, enter matrix **C**.

We are ready now to evaluate the matrix problem $2A - 3(B - C)$. Begin by entering 2. Place an **A** beside it using **2ND | MATRIX | Names | 1: [A] ENTER**.

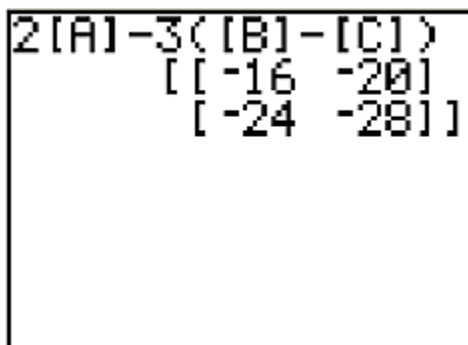
At this point we have **2A** on the screen. Next, enter **-3(** and the display should read **2A - 3(**.

B is entered with **2ND | MATRIX | Names | 2: [B] | ENTER** as shown here.



NAME	MATH	EDIT
1: [A]	2x2	
2: [B]	2x2	
3: [C]	2x2	
4: [D]		
5: [E]		
6: [F]		
7↓ [G]		

Finally, enter **- [C])** and press **ENTER**.



2[A]-3([B]-[C])
 [[-16 -20]
 [-24 -28]

The final matrix answer is displayed.



Calculator Appendix R



Advanced Matrix Math: Multiplication, Inverse, Determinant

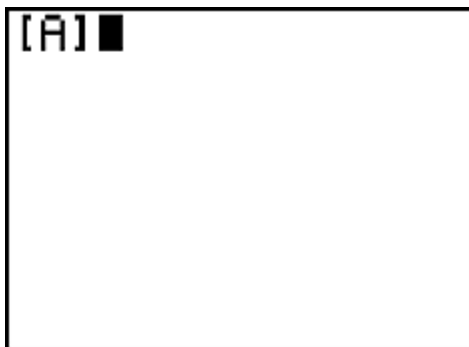
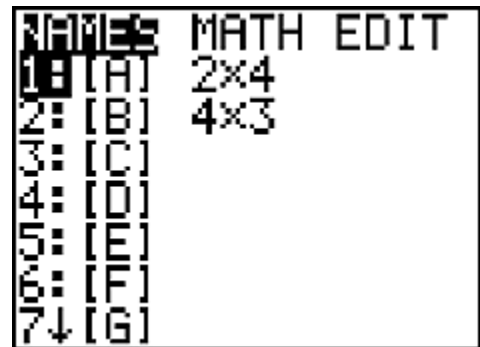
Matrices A and B can be **multiplied** in the order, AB , only if the number of columns in A is equal to the number of rows in B .

As an example, let $A = \begin{bmatrix} 1 & 3 & 2 & -1 \\ 2 & 1 & 0 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 4 & 1 \\ 3 & 1 & 2 \\ 1 & 1 & 3 \end{bmatrix}$

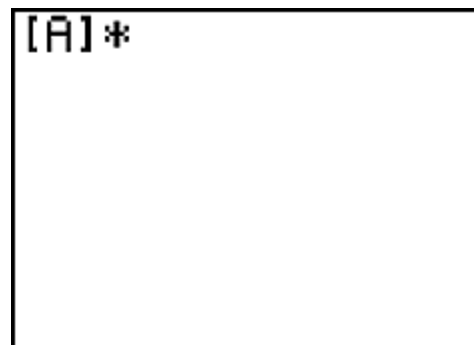
As described in **Calculator Appendix Q**, enter the A and B matrices.



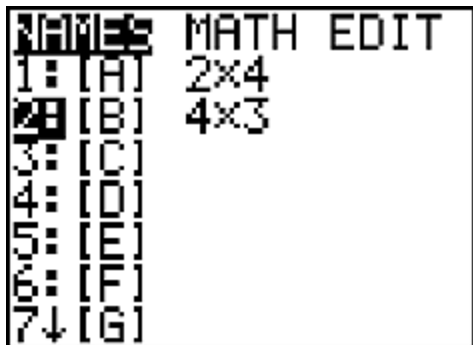
Using **2nd MATRIX**, select the A matrix and then press **ENTER**.



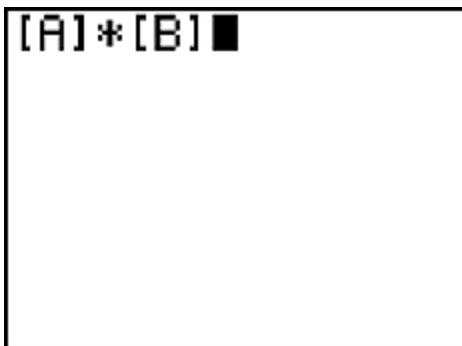
The resulting main screen display is shown here.



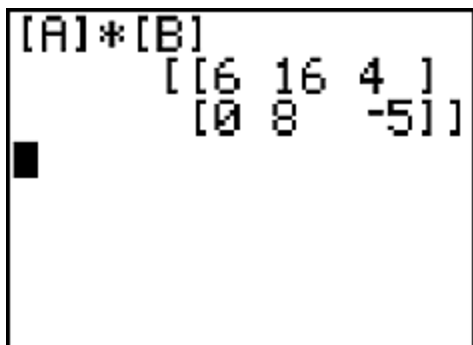
We want to multiply this by the B matrix, so enter a times sign.



Using **2nd MATRIX | DOWN ARROW**, select the *B* matrix and press **ENTER**.



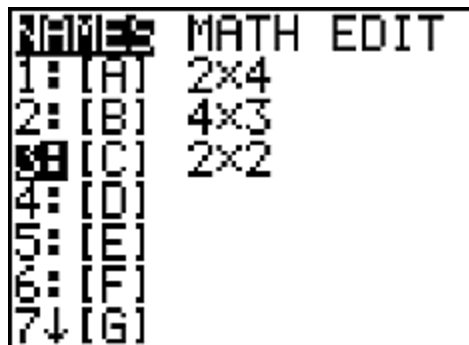
The main display will now look like this.



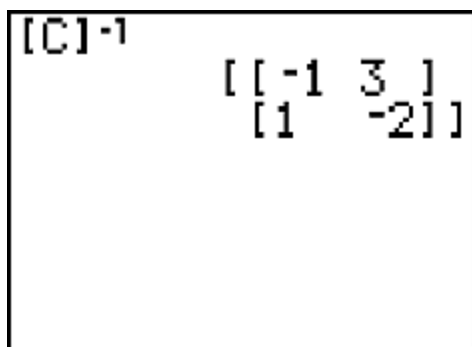
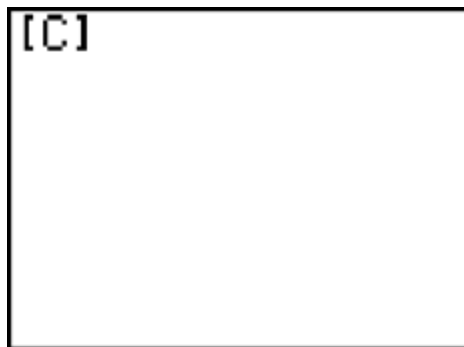
Press **ENTER** to obtain the final answer matrix.

To find the **inverse** of, for example, matrix $C = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$, first enter matrix *C* as described in **Calculator Appendix Q**.

Use **2nd MATRIX** and the **DOWN ARROW** to select matrix *C*.



Press ENTER and obtain this display.



The inverse of C is represented symbolically by raising C to the -1 power.

Press the X^{-1} key and then **ENTER** to obtain the final answer, a matrix that is the inverse of C .

To find the **determinant** of, for example, matrix $C = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$, first enter matrix C as described in **Calculator Appendix Q**.

Use **2nd MATRIX | MATH | 1: det(| ENTER** to prepare to take the determinate of a matrix .



The main screen now displays this:

At this point we need to put the matrix C in the parenthesis.

```
det(
```

NAME	MATH EDIT
1: [A]	2x4
2: [B]	4x3
3: [C]	2x2
4: [D]	
5: [E]	
6: [F]	
7↓ [G]	

Using **2nd MATRIX | DOWN ARROW**, select the C matrix and then press **ENTER**.

Close the parenthesis and press **ENTER** to produce the answer.

```
det([C])           -1
```



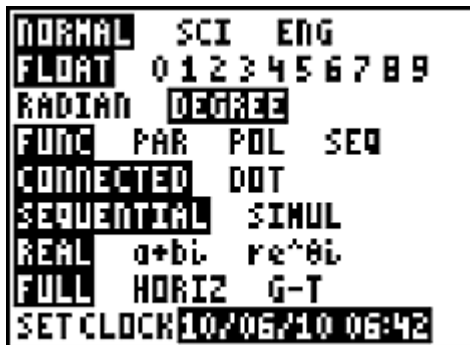
Calculator Appendix S



Angle conversions: (degrees, minutes, seconds, radians)

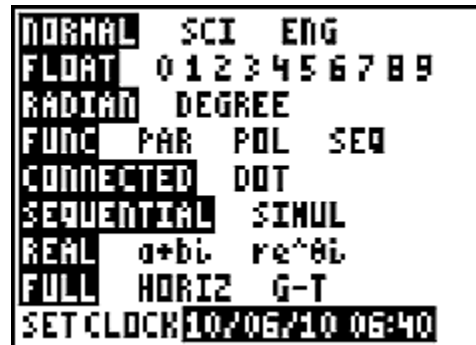
Changing Modes:

When doing angle conversions or evaluating trig expressions it is important to be in the correct angle mode (degree or radians). To verify which mode is currently set or to change the mode, press the **MODE** button.



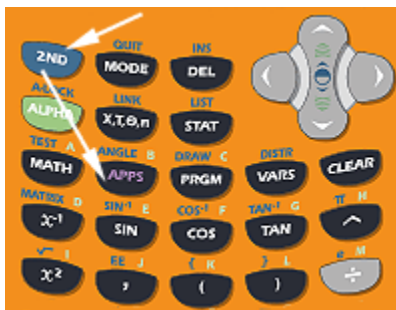
In the adjacent picture notice that the angle mode is **DEGREE**. To change to **RADIAN** press the **DOWN ARROW** twice, and then **ENTER**.

The result will be as shown just below.



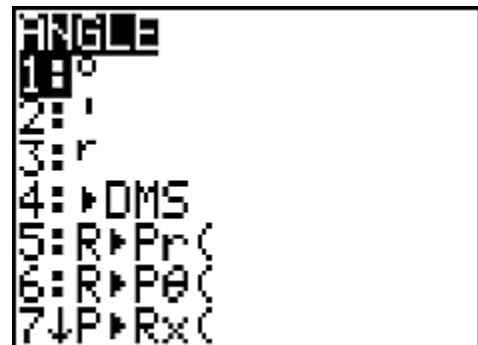
Conversion from DMS to decimal degrees:

Make sure **DEGREE** mode is set. Enter $127^{\circ} 47' 22''$ (read as 127 degrees, 47 minutes, and 22 seconds) and we wish to convert to decimal degrees (127.7894444).



First enter 127. Produce the degree symbol ($^{\circ}$) by pressing **2nd ANGLE**. The degree symbol ($^{\circ}$)

is labeled as **1: $^{\circ}$** . Press **ENTER** and the display should now read 127° .

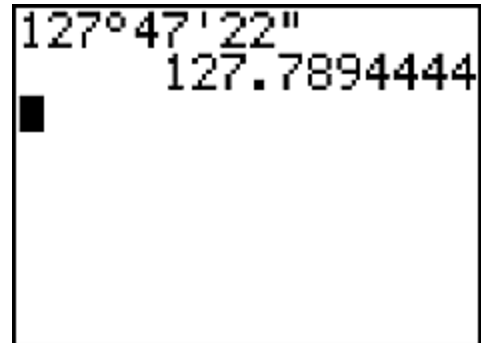
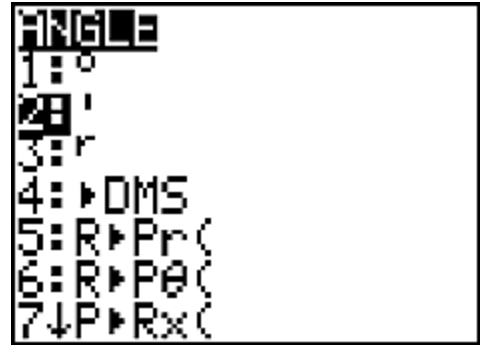


Next, enter 47. Produce the minute symbol (') by pressing **2nd ANGLE** and then choosing **2: '** . The display should now read $127^{\circ} 47'$.

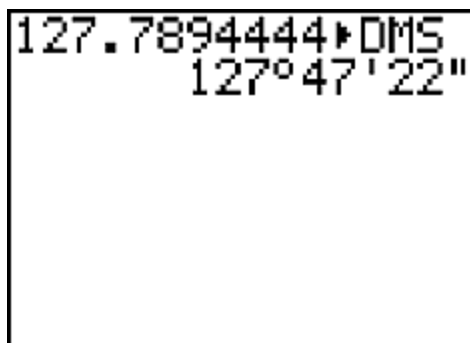
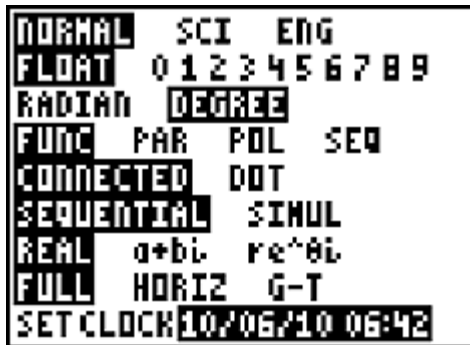


Enter 22 and then the second symbol

("). With the display now reading $127^{\circ} 47' 22''$, press **ENTER** and the decimal equivalent (127.7894444) should appear.



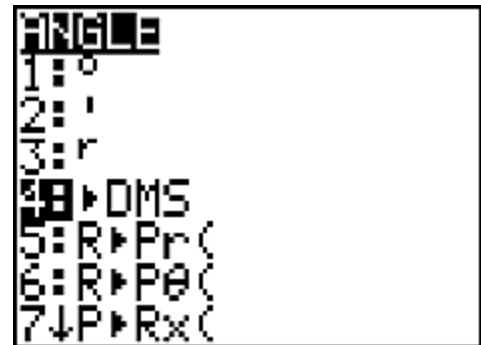
Conversion from decimal deg to deg, min, sec:



Use the **MODE** button to make sure the angle mode is **DEGREE**. Then enter

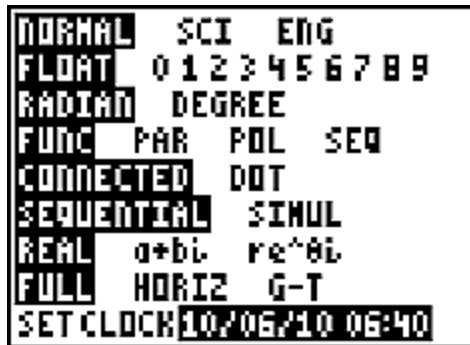
127.7894444 as a decimal degree entry.

Using **2nd ANGLE** choose **4: DMS**.



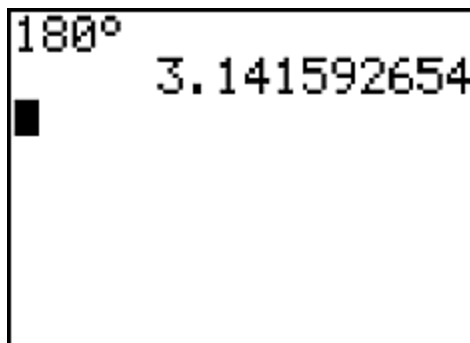
Press **ENTER** and the answer in degrees, minutes, and seconds should appear.

Conversion from degree to radians:



Using the **MODE** button, set the angle mode to **RADIAN**.

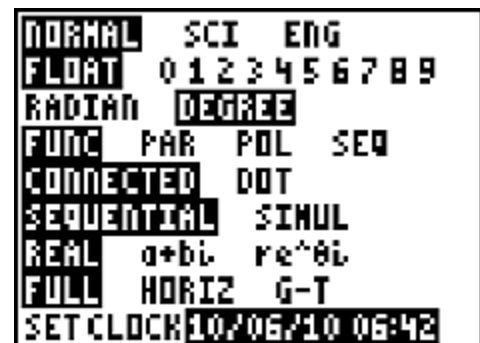
Next, enter the desired degree value (180 for our example). Since we are currently in radian mode it is necessary to specify that 180 is really a degree value. Do this with 2nd **ANGLE** | 1: °.



Press **ENTER** and the equivalent radian value will be displayed.

Conversion from radians to degrees:

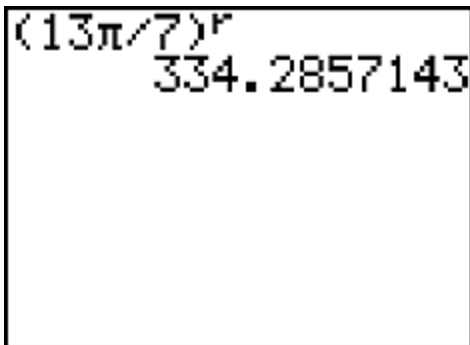
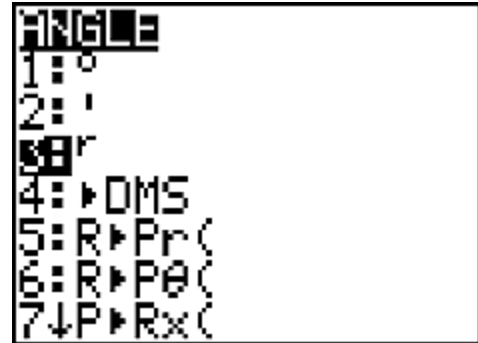
Using the **MODE** button, set the angle mode to **DEGREE**.



Enter the radian value ($13\pi/7$ for our example). It is important to enter this expression **in parenthesis** since math is involved (division by 7).

Since the current angle mode is degrees, it is necessary to specify that $13\pi/7$ is really a radian value by using a superscript r ... $(13\pi/7)^r$.

This superscript r is provided with **2nd ANGLE**.



Press **ENTER** and the equivalent degree answer will be displayed.



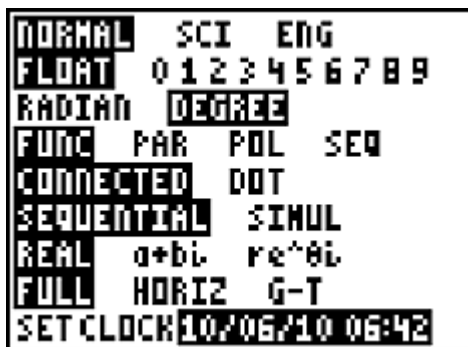
Calculator Appendix T

(Evaluation of trig expressions)



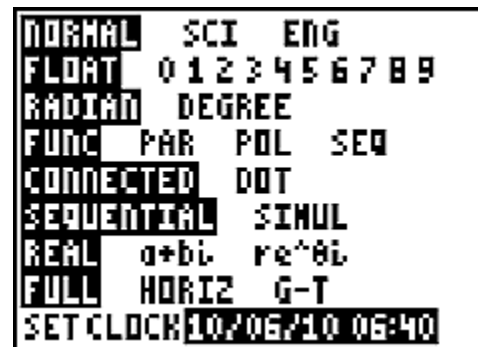
Changing Modes:

When doing angle conversions or evaluating trig expressions it is important to be in the correct angle mode (degree or radians). To verify which mode is currently set or to change the mode, press the **MODE** button.



In the adjacent picture notice that the angle mode is **DEGREE**. To change to **RADIAN** press the **DOWN ARROW** twice, and then **ENTER**.

The result will be as shown here. The radian mode is now set. At this point pressing the **RIGHT ARROW** and then **ENTER** would cause the **DEGREE** mode to be set.



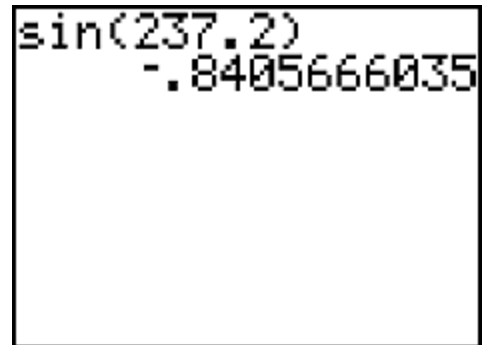
Only the three main trig functions (sin, cos, & tan) are directly available on a graphing calculator. To obtain the other three, use these identities:

$$\begin{aligned} \csc(\theta) &= 1/\sin(\theta) \\ \sec(\theta) &= 1/\cos(\theta) \\ \cot(\theta) &= 1/\tan(\theta) \end{aligned}$$



Example 1:

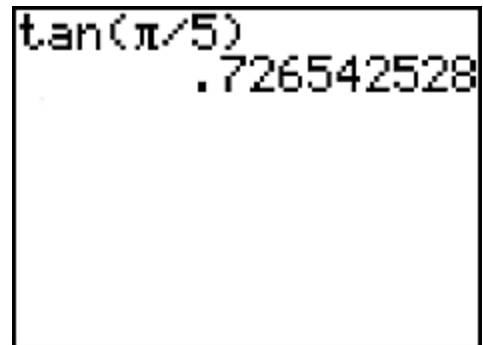
To find the sine of 237.2 degrees, use the **MODE** button to make sure the **DEGREE** mode is set. Then press the **SIN** button. The screen will now display **sin(** after which 237.2 can be entered. Close the parenthesis and press **ENTER**. The answer, $-.8405666035$, is displayed.



A calculator screen showing the calculation of the sine of 237.2 degrees. The display shows "sin(237.2)" on the first line and the result "-.8405666035" on the second line.

Example 2:

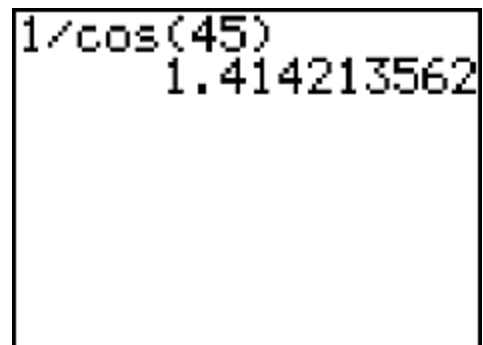
To find the tangent of $\pi/5$ radians, use the **MODE** button to make sure the **RADIAN** mode is set. Then press the **TAN** button. The screen will now display **tan(** after which $\pi/5$ can be entered. Close the parenthesis and press **ENTER**. The answer, $.726542528$, is displayed.



A calculator screen showing the calculation of the tangent of $\pi/5$ radians. The display shows "tan($\pi/5$)" on the first line and the result ".726542528" on the second line.

Example 3:

To find the secant of 45 degrees, use the **MODE** button to make sure the **DEGREE** mode is set. Using the identity $\sec(\theta) = 1/\cos(\theta)$, enter **1/** and then press the **COS** button. The screen will now display **1/cos(** after which 45 can be entered. Close the parenthesis and press **ENTER**. The answer, 1.414213562 , is displayed.



A calculator screen showing the calculation of the secant of 45 degrees using the identity $\sec(\theta) = 1/\cos(\theta)$. The display shows "1/cos(45)" on the first line and the result "1.414213562" on the second line.

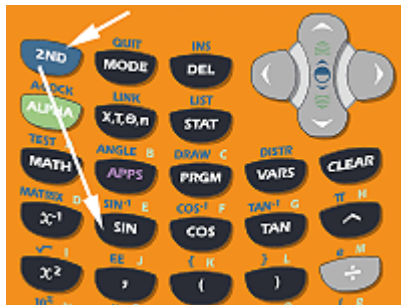


Calculator Appendix U



(Inverse trig functions)

Quite often we may know the sine of an angle but not the angle itself. To find the angle use the SIN^{-1} function (read this as “inverse sine”).



Suppose we know that $\sin \theta = .78382$ and we wish to know θ .

Enter 2^{nd} SIN^{-1} .

At this point the display shows sin^{-1} (inviting us to enter the argument, .78382. Do so, close the parenthesis, and press **ENTER**.

```
sin-1(.78382)
51.61167402
```

The answer shown here is the value of the angle θ in degrees. A degree answer was produced here because the angle mode was set to **DEGREE**. To produce a radian answer use the **MODE** key to change the mode to **RADIAN**. (See **Calculator Appendix S** for how to change the angle mode.)

Similarly, if we know $\cos \theta = .56203$, find θ with 2^{nd} COS^{-1} .

If we know $\tan \theta = 1.5592$, find θ with 2^{nd} TAN^{-1} .



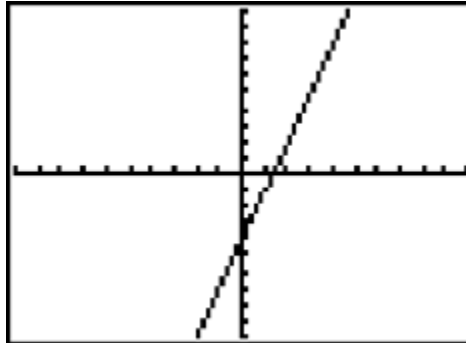
Calculator Appendix V



Applying absolute value to a function... $|f(x)|$ & $f(|x|)$

Consider the linear function $y = 3x - 4$. When entered as **Y1** the graph is as shown below:

```
Plot1 Plot2 Plot3
\Y1=3X-4
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```



We want to investigate what happens to the graph when the absolute value is applied in different places:

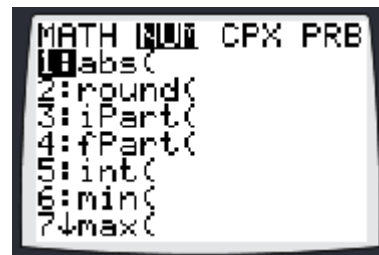
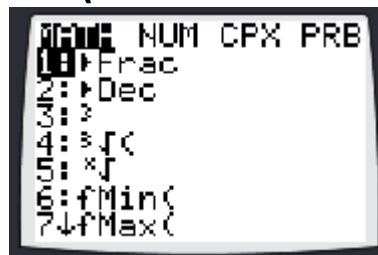
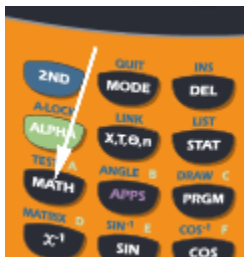
$$y = 3|x| - 4$$

$$y = |3x - 4|$$

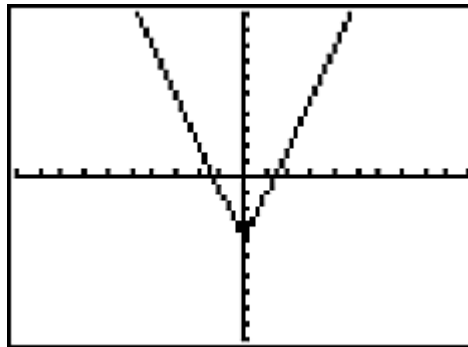
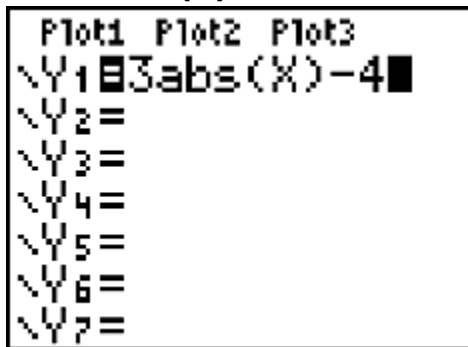
Enter the function $3|x| - 4$ for **Y1**. Unfortunately, the vertical bars that are traditionally used to indicate absolute value are not available on the calculator. The function must be entered as **Y1 = 3abs(X) - 4**

Access the absolute value function (given by **abs** above) with:

Math | Num | 1: abs(



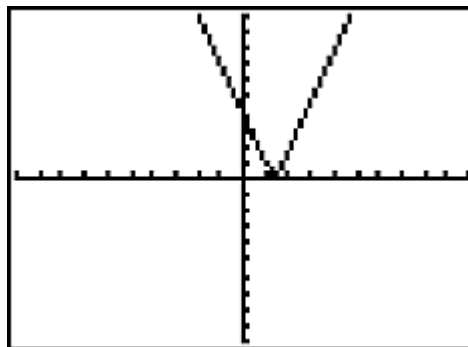
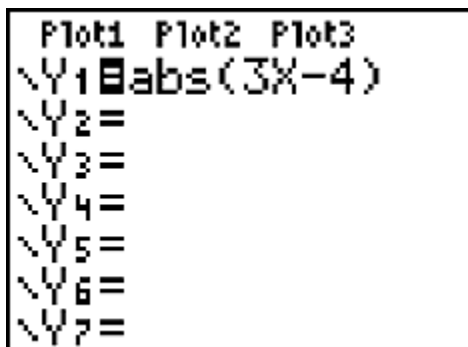
With $3\text{abs}(X) - 4$ entered for Y_1 , press **GRAPH** (Use **6: ZStandard** zoom)



Notice that this graph is produced with the following rule:

Delete the graph of the original function to the left of the y-axis. Then mirror image the remaining function across the y-axis to replace what was originally on the left side.

Now consider the function $Y_1 = \text{abs}(3x - 4)$ and its graph:



Notice that this graph is produced with the following rule:

Replace any portion of the original graph below the x-axis with its mirror image across the x-axis.



Calculator Appendix W

(Graphing the sine function)



We will graph $f(x) = \sin(x)$ where x is in radians. First make sure the **MODE** is in radians... covered in **Appendix S**.



Press the **Y=** button and prepare to enter our function as **Y1**. Press the **SIN** button and **sin(** will appear on the screen. Enter **X** and close the parenthesis.

```
Plot1 Plot2 Plot3
\Y1=sin(X)
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

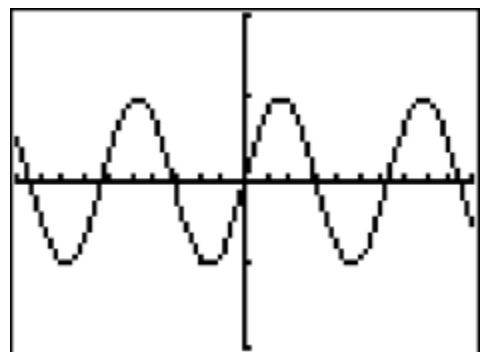
The correctly entered function will appear like this:

```
WINDOW
Xmin=-10
Xmax=10
Xscl=1
Ymin=-2
Ymax=2
Yscl=1
Xres=1
```

Press the **WINDOW** key and make these entries:

Press the **GRAPH** button to produce this resulting graph:

Experiment by changing the function to $\sin(x) + 1$, $2\sin(x) + 1$, $-2\sin(x) + 1$, etc.





Calculator Appendix X

(Graphing the cosine function)



We will graph $f(x) = \cos(x)$ where x is in radians. First make sure the **MODE** is in radians... covered in **Appendix S**.



Press the **Y=** button and prepare to enter our function as **Y1**. Press the **COS** button and **cos(** will appear on the screen. Enter **X** and close the parenthesis.

```
Plot1 Plot2 Plot3
\Y1=cos(X)
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

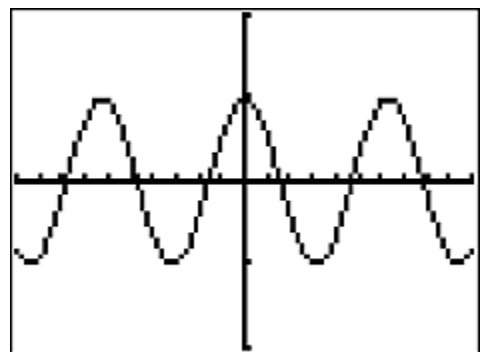
The correctly entered function will appear like this:

```
WINDOW
Xmin=-10
Xmax=10
Xscl=1
Ymin=-2
Ymax=2
Yscl=1
Xres=1
```

Press the **WINDOW** key and make these entries:

Press the **GRAPH** button to produce this resulting graph:

Experiment by changing the function to $\cos(x) + 1$, $2\cos(x) + 1$, $-2\cos(x) + 1$, etc.



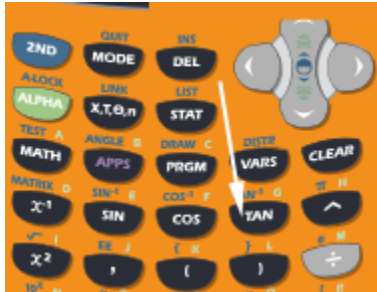


Calculator Appendix Y

(Graphing the tangent function)



We will graph $f(x) = \tan(x)$ where x is in radians. First make sure the **MODE** is in radians... covered in **Appendix S**.



Press the **Y=** button and prepare to enter our function as **Y1**. Press the **TAN** button and **cos(** will appear on the screen. Enter **X** and close the parenthesis.

The correctly entered function will appear like this:

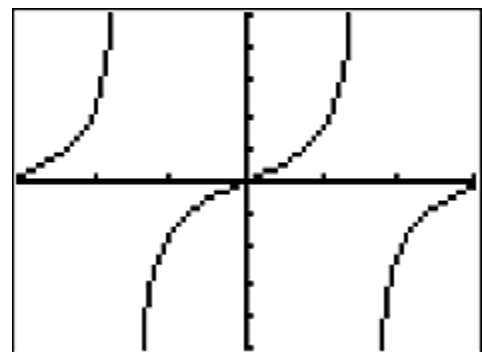
```
Plot1 Plot2 Plot3
\Y1=tan(X)
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

```
WINDOW
Xmin=-3
Xmax=3
Xscl=1
Ymin=-5
Ymax=5
Yscl=1
Xres=1
```

Press the **WINDOW** key and make these entries:

Press the **GRAPH** button to produce this resulting graph:

Experiment by changing the function to $\tan(x) + 1$, $2\tan(x) + 1$, $-2\tan(x) + 1$, etc.





Calculator Appendix Z

(Greatest integer function)

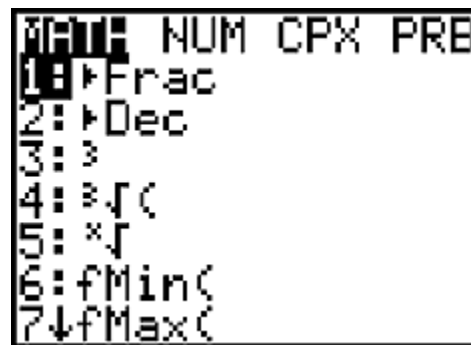


The greatest integer function is typically written as $f(x) = \lfloor x \rfloor$. This yields the largest integer that is less than or equal to x . For example, $\lfloor 2.3 \rfloor = 2$ and $\lfloor -2.3 \rfloor = -3$.



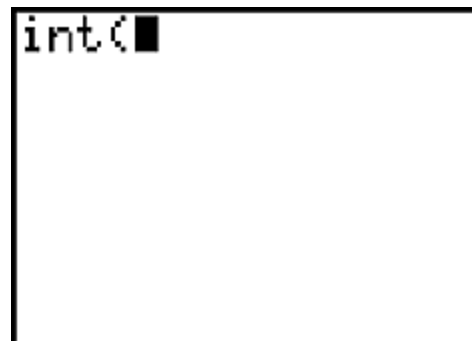
To work $\lfloor 2.3 \rfloor$ on the calculator press **MATH**.

This is the resulting screen. Use the **RIGHT ARROW** to move across to **NUM**.

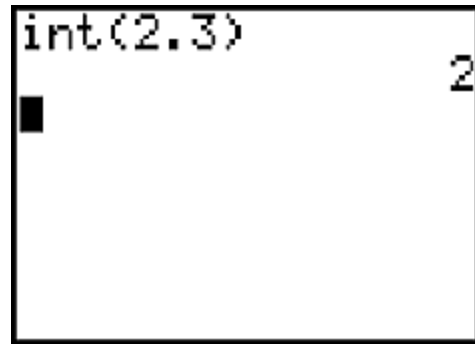


Either press **5** or **DOWN ARROW** to **5:int(**.

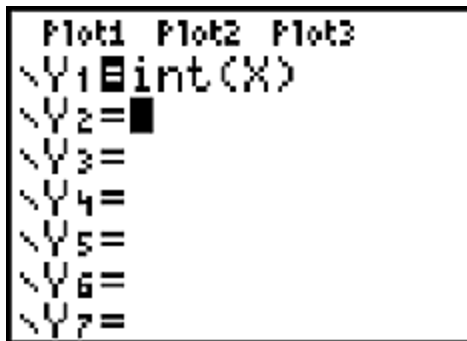
This will appear on the screen inviting us to enter an argument.



Enter **2.3** and then press **ENTER**. The answer **2** will appear.



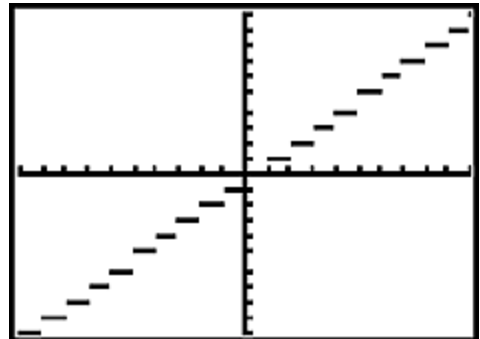
int(2.3) 2



Plot1 Plot2 Plot3
 \Y1=int(X)
 \Y2=
 \Y3=
 \Y4=
 \Y5=
 \Y6=
 \Y7=

To graph the greatest integer function, press **Y=** | **MATH** | **NUM** | **5:int(** . Then enter **X** and close the parenthesis.

Press **GRAPH** to produce this classical step function. Although not shown on the graph, think of each little horizontal line segment as having a **solid dot** on the left end and an **open circle** on the right end.





Calculator Appendix AA



(Sum of a series)

Consider the sum given by $\sum_{k=-2}^8 \left(\frac{1}{k^2+2} \right)$:

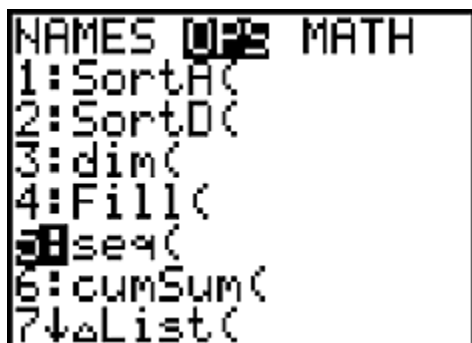


To obtain the solution to this on the graphing calculator, press **2nd LIST**, **RIGHT ARROW** across to **MATH**, and then **DOWN ARROW**

ARROW down to **5: sum(**.

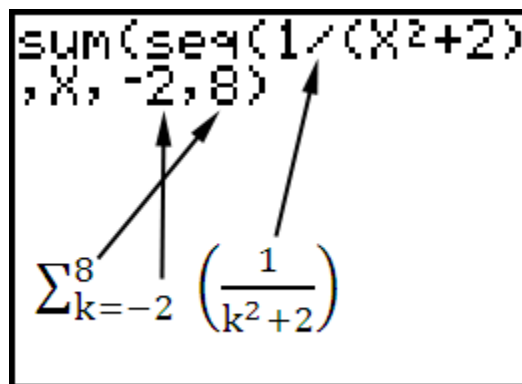
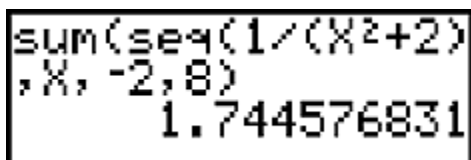
At this point we would have **sum(** on the display.

Press **2nd LIST**, **RIGHT ARROW** across to **OPS**, and **DOWN ARROW** down to **5: seq(**.



At this point we should have **sum(seq(** on the display and we are now ready to enter our problem as follows:

Press **ENTER** and the answer appears:





Calculator Appendix AB

(Graphing parametric equations)

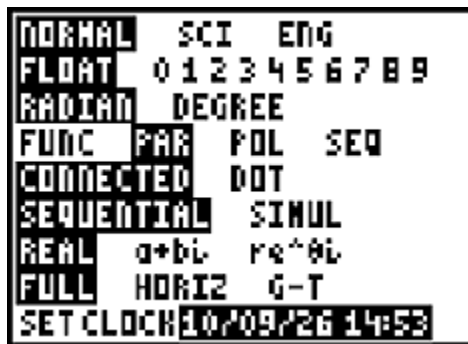


Consider the pair of parametric equations given by:

$$x = t$$

$$y = t^2$$

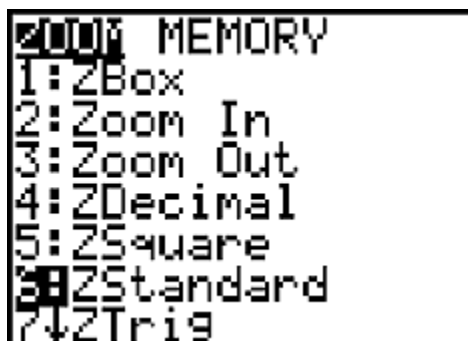
The parameter t will be restricted to $\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$.



To graph this pair, first put the calculator in Parametric mode. Press the mode button and then with the **RIGHT ARROW** and **DOWN ARROW**, move down to **PAR** and press **ENTER**.



Now enter the pair of equations by pressing the **Y=** button. Enter t for **X1=** and t^2 for **Y1=**.



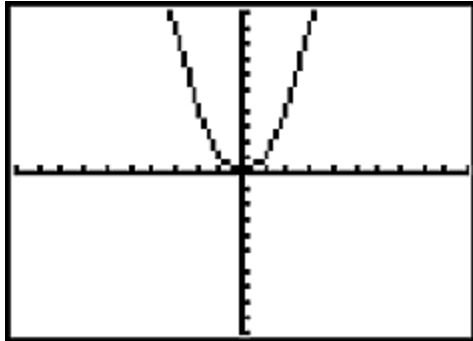
Next, Use the **ZOOM** button to choose **6: ZStandard**

And now, most critical of all, we use the **WINDOW** button to set the **Tmin**, **Tmax**, and **Tstep** values so as to use only the desired values of, $\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

```

WINDOW
Tmin=-4
Tmax=4
Tstep=1
Xmin=-10
Xmax=10
Xscl=1
↓Ymin=-10

```



Finally, press **GRAPH** to produce this picture:

Although the graph produced by the calculator may appear to be a smooth curve it is in reality a series of line segments connecting the individual (x, y) points corresponding to the individual t values as shown by the following table.

t	$x(t) = t$	$y(t) = t^2$
-4	-4	16
-3	-3	9
-2	-2	4
-1	-1	1
0	0	0
1	1	1
2	2	4
3	3	9
4	4	16

It must be realized that t values determine the domain and range of the $y = y(x)$ function. Notice in the table above that the largest y value is 16. Using the **WINDOW** button, change **Ymax** to 20. The resulting parabola graphed can be now seen to abruptly stop at the y value of 16 with nothing drawn above that value.



Calculator Appendix AC

(Graphing in polar coordinates)

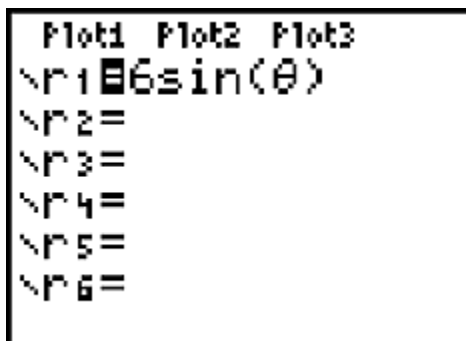


Consider the polar coordinates equation given by $r = 6\sin(\theta)$.



To graph this in polar coordinates, first put the calculator in polar mode. Do this by pressing the **MODE** button.

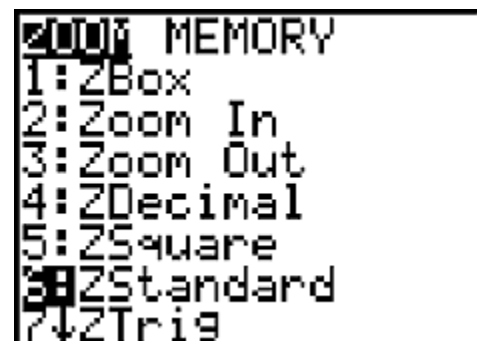
With the **DOWN** and **RIGHT ARROWS** select **POL** and press **ENTER**. It is possible to be in either **RADIAN** or **DEGREE** mode. Notice here that we are in **DEGREE** mode.



Press **Y=** and enter $r1 = 6\sin(\theta)$.

To make this function graph properly, it is necessary to set the **ZOOM** level and also to make adjustments to the **WINDOW** settings.

Press **ZOOM** and choose **6: ZStandard**.



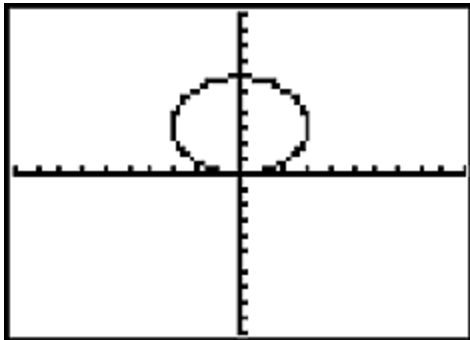
```

WINDOW
θmin=0
θmax=360
θstep=2
Xmin=-10
Xmax=10
Xscl=1
↓Ymin=-10

```

Next, press **WINDOW** and make the settings shown here. **θmin**, **θmax**, and **θstep** are especially important. Notice that we go through a full 360 degrees since the **DEGREE** mode was previously set. If the **RADIAN** mode was set it would be appropriate for θ to advance from 0 to 2π .

In general, it is appropriate to advance θ through a full cycle of the trig function present in the equation.



Finally press **GRAPH** to produce the graph of the polar equation.



Calculator Appendix AD



Binomial Expansion Coefficients

Consider the expansion of

$$(a + b)^6 = \sum_{r=0}^6 {}_6C_r a^{6-r} b^r$$

This requires the calculation of ${}_6C_0$, ${}_6C_1$, ${}_6C_2$, ${}_6C_3$, ${}_6C_4$, ${}_6C_5$, and ${}_6C_6$. If you are not familiar with combinations, see **Calculator Appendix O**.

Produce these combination values automatically on the graphing calculator as follows:



Press **STAT, 1:Edit**.

Under the **L1** column enter 0, 1, 2, 3, 4, 5, and 6.

L1	L2	L3	2
0			
1			
2			
3			
4			
5			
6			
L2(1)=			

Using the **LEFT** and **UP ARROWS** move the cursor on top of the **L2** column.

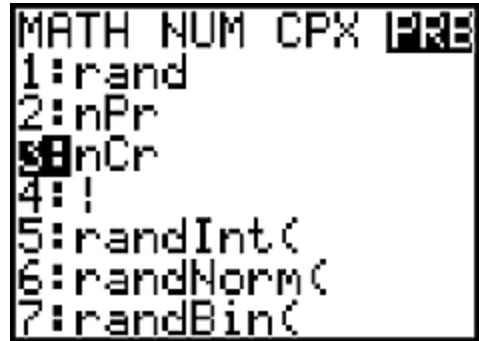
At this point we are going to enter “6 nCr L1”. Here’s how we do this:

First, enter 6.

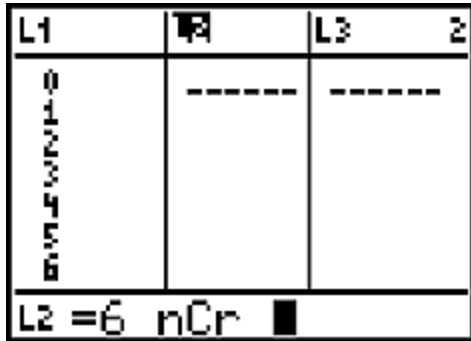
L1	L2	L3	2
0			
1			
2			
3			
4			
5			
6			
L2 = 6			



Press **MATH** and then **RIGHT ARROW** across to **PRB**.



Choose **3:nCr**.

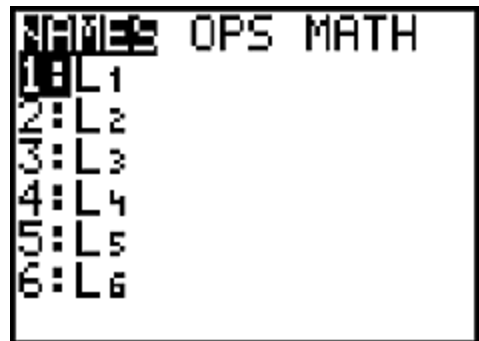


The display should now look like this.

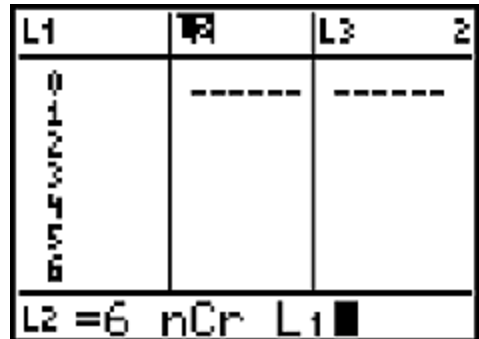


All we need to do now is enter L1. Do this by pressing **2nd LIST**.

Choose **1.L1**.



Now the display looks like this and we are almost finished.



Move down one level with the **DOWN ARROW** and the answers for our combinations will automatically appear in the **L2** column.

L1	L2	L3	Z
0	1	-----	
1	6		
2	15		
3	20		
4	15		
5	6		
6	1		
L2(1)=1			

Here is the meaning of the numbers in the L2 column:

$${}^6C_0 = 1$$

$${}^6C_1 = 6$$

$${}^6C_2 = 15$$

$${}^6C_3 = 20$$

$${}^6C_4 = 15$$

$${}^6C_5 = 6$$

$${}^6C_6 = 1$$



Calculator Appendix AE



Using Table Set and Table to show the limit of a function

Consider the following “limit” expression:

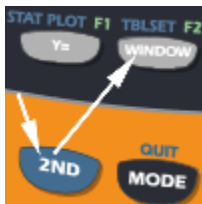
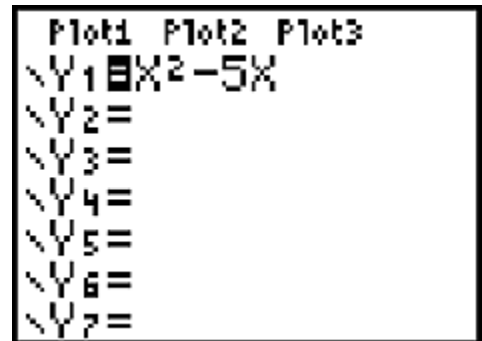
$$\lim_{x \rightarrow 2^-} (x^2 - 5x)$$

To show values of x approaching 2 from the left (and the resulting function values), we need to produce the adjacent table on a graphing calculator.

x	$f(x) = x^2 - 5x$
1.5	-5.25
1.6	-5.44
1.7	-5.61
1.8	-5.76
1.9	-5.89
2.0	-6.0



Click on **Y=** and enter the function $X^2 - 5X$.



Use **2nd TBLSET** to set up the table. Basically, we want to tell it to start with an x value of 1.5 and with increments of .1. The second column of the resulting table will be the corresponding function value (the **Y1** function above).

Set **TblStart** to our initial x value of 1.5
 Set **ΔTbl** to our increment value of .1.
 Make sure **Indpnt(x)** and **Depend(y)** are in the **Auto** mode.





Use **2nd TABLE** to actually produce the table.

Notice that as x approaches 2 (1.5, 1.6, 1.7, 1.8, 1.9) the values of the function seem to be approaching -6 .

X	Y ₁	
1.5	-5.25	
1.6	-5.44	
1.7	-5.61	
1.8	-5.76	
1.9	-5.89	
2	-6	
2.1	-6.09	
X=1.5		



Calculator Appendix AF



Derivatives evaluated at particular location

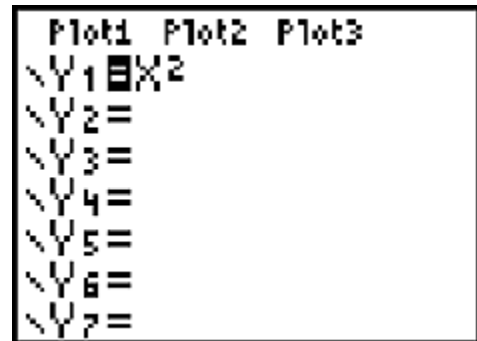
Consider the function $f(x) = x^2$. The TI 84 calculator is **not** capable of telling us that $f'(x) = 2x$; however, it **is** capable of evaluating the derivative at a particular x value. For $x = 5$ the, calculator will yield an answer of 10 for the derivative.

There are two ways to calculate a numerical derivative (For the sake of the examples below, assume the function is x^2 for which we want the derivative):

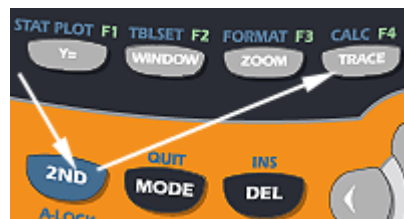
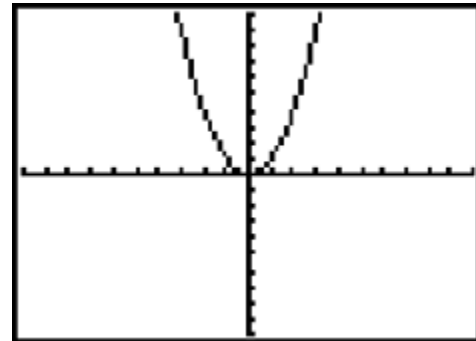
Technique #1(using the graph of the function):



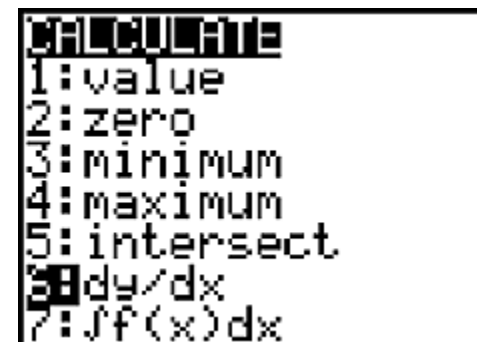
Use **Y=** to enter **Y1 = x^2** .



Then press the **GRAPH** button to produce this picture.



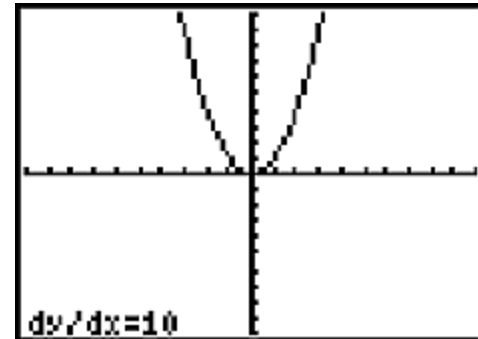
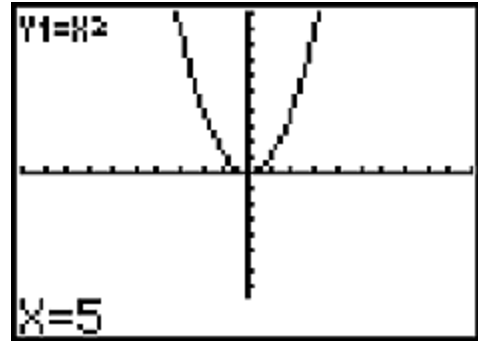
Use **2ND CALC** to bring up this menu. Choose **6: dy/dx** .



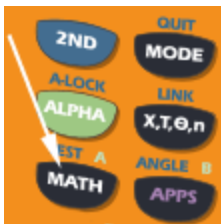
The screen display will now show the graph. Enter **5** and the display will appear as shown here.

Note that only x values within the span of the window are permitted.

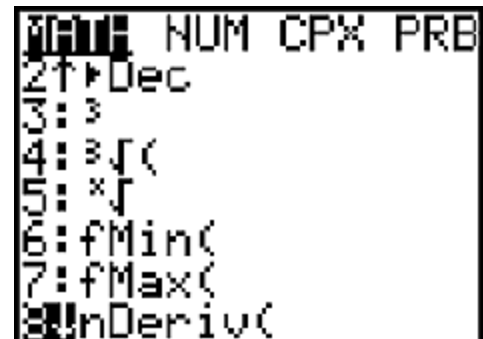
Finally, press **ENTER** for the answer (10) for the derivative of x^2 evaluated at $x = 5$.



Technique # 2

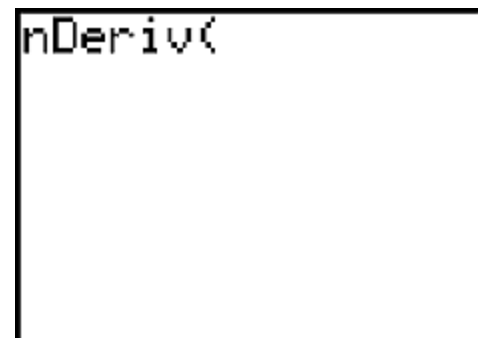


Press **Math** | **Math** |
8: nDeriv(

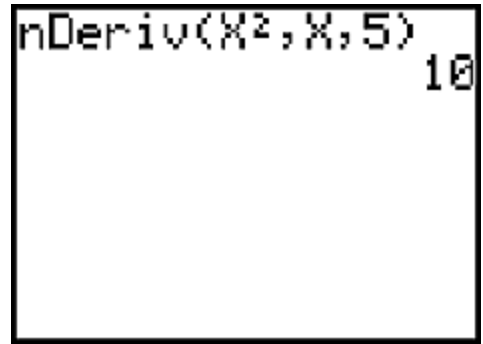


The screen display will appear as shown here. At this point we are being invited to enter 3 parameters.

- The function for which we are taking the derivative (x^2)
- X (we are taking the derivative with respect to x)
- The x value at which the derivative is to be evaluated (5).



For our problem, enter the parameters as shown here and then press **ENTER** to produce the final answer (10).

A rectangular box representing a calculator screen. The text inside the box is "nDeriv(X^2,X,5)" on the top line and "10" on the second line, positioned to the right of the first line.

Additional notes for Technique #2:

There is actually an optional fourth parameter for **nDeriv**. By default it is .001. The calculator goes through an incremental process (loop) in calculating the ratio of differences in order to produce the derivative. This parameter is used as the incremental x value in that process. By making this value smaller, the accuracy of the answer can be improved. This is especially helpful when the derivative is desired at very steep positions on the curve.

Instead of entering the function directly as shown above (x^2), use **VAR** | **Y-Vars** | **1: Function** to directly select **Y1**, **Y2**, etc. This is helpful as a time saver if an especially tedious function has been previously entered as one of the **Y** values.



Calculator Appendix AG



Graphing indefinite integrals

The TI-84 is **not** capable of receiving a function and then **returning a new function** that is the indefinite integral of the original function.

It **is**, however, capable of being given a function and then **graphing one member of its indefinite integral family**.

Consider the integral $\int x^2 dx = \frac{x^3}{3} + C$

The entry of this indefinite integral into the **y=** dialogue of the calculator is a bit awkward since it is typically entered using two variables.



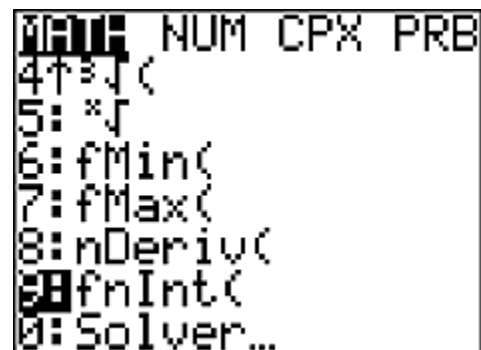
Press the **y =** button. What we want to enter for **Y1=** is **fnInt(T²,T,0,X)**.

At first glance this does not look like the problem $\int x^2 dx$. The function x^2 is to be entered as T^2 . The T that follows indicates integration with respect to T . The 0 is the lower limit and X is the upper limit from the following representation of the Fundamental Theorem of Calculus where $F(x)$ is the indefinite integral of $f(x)$.

$$\int_0^x f(T) dT = F(x) - F(0)$$



To enter the **fnInt(** part, press the **Math** button and then choose **9: fnInt(** .



Thus far the entry for $Y_1 =$ appears as shown here.

```

Plot1 Plot2 Plot3
\Y1=fnInt(
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=

```



Enter T^2 with this sequence of key strokes. After T^2 , enter $,T,0,X$ using the X,T,θ,n button to enter the X .

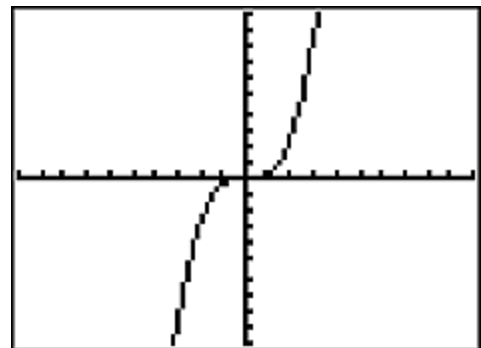
The display will now appear like this.

```

Plot1 Plot2 Plot3
\Y1=fnInt(T^2,T,0,X)
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=

```

Press **Graph** to produce this final display of the graph of one member of the indefinite integral family of $\int x^2 dx$. Be prepared to wait. It is very slow to display.



Incidentally, when we entered $\text{fnInt}(T^2,T,0,X)$ for $Y_1=$, we could have entered $\text{fnInt}(X^2,X,0,X)$, instead; however, this can be confusing for students and the T version is generally considered best.



Calculator Appendix AH



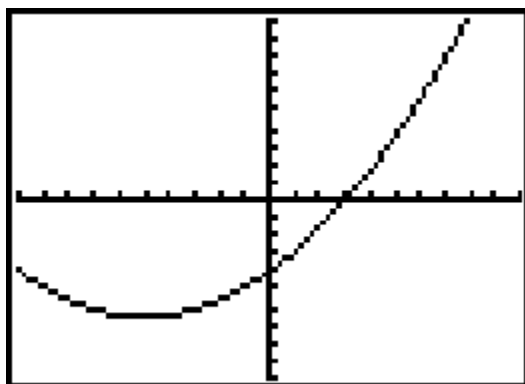
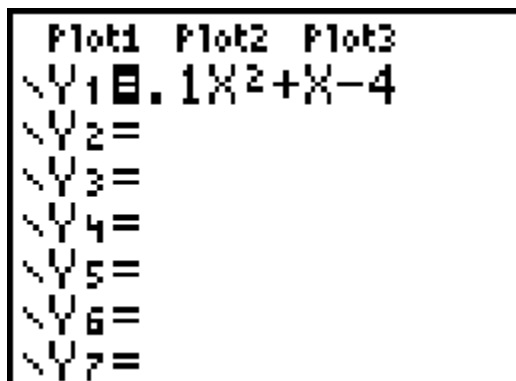
Definite Integrals on a Graphing Calculator

There are two ways to do definite integrals on a graphing calculator:

First method:

For example, to evaluate

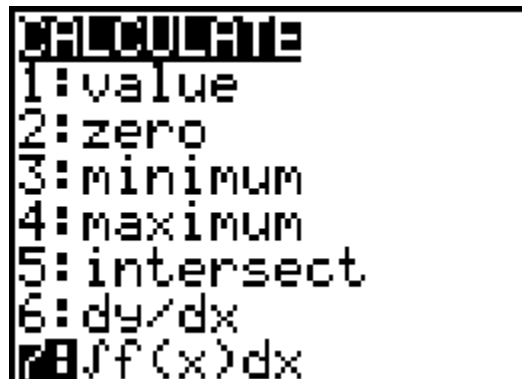
$\int_{-2}^7 (.1x^2 + x - 4)dx$, enter the function as follows using the **Y=** button.



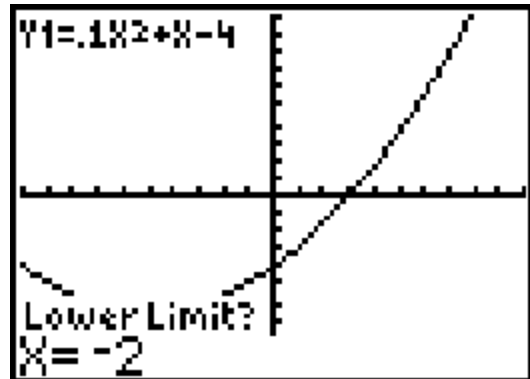
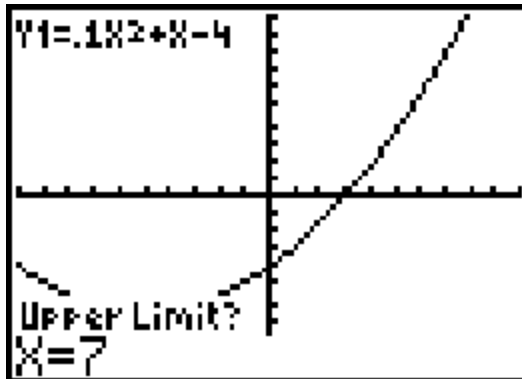
Graph the function by pressing the **GRAPH** button.



Press **2nd CALC | 7: ∫ f(x)dx** and then press **ENTER**.

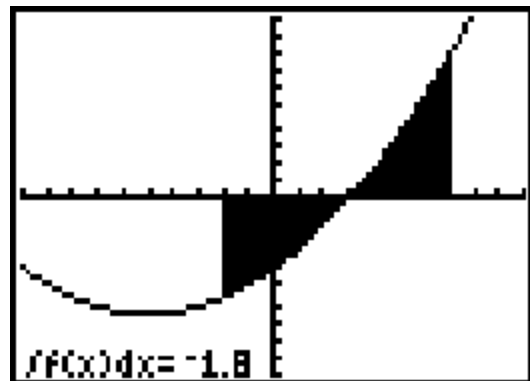


Enter -2 for the lower limit and press **ENTER**.



Enter 7 for the upper limit and press **ENTER**.

The answer to this definite integral is -1.8 . Notice the shaded area that represents the area “under the curve” between the limits of -2 and 7 .

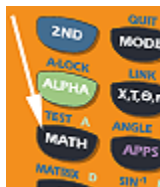


This method has the **disadvantage** of requiring that the **two limits be within the zoom area**.

Second method:

To demonstrate this method, use the same integral as with the previous method: $\int_{-2}^7 (.1x^2 + x - 4)dx$

Press the **MATH** button.



The screen shown here appears. Either press **9** or **DOWN ARROW** down to it. Press **ENTER**.

```

NUM CPX PRB
3: ↑
4: ∫(
5: *∫
6: fMin(
7: fMax(
8: nDeriv(
9: fnInt(

```

```

fnInt(.1X^2+X-4,X
,-2,7)

```

fnInt(will appear on the screen. Enter the definite integral as shown here and press **ENTER**.

The answer is -1.8 .

```

fnInt(.1X^2+X-4,X
,-2,7)
-1.8

```

Suppose the function was already entered as **Y1=** with the **Y=** button. Instead of entering the function directly as was demonstrated above, **Y1** can be entered with this button sequence:

VARS | Y-VARS | 1: FUNCTION | Y1



Calculator Appendix AI

(Newton's Method)



Use three iterations of Newton's method to find an estimate of the irrational root of $f(x) = x^3 + 2x - 4$ that lies somewhere between 1 and 2.

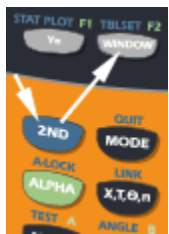
Newton's method for finding a better estimate (x_1) of a root of $f(x)$ in terms of an initial estimate (x_0).

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = x_0 - \frac{x_0^3 + 2x_0 - 4}{3x_0^2 + 2}$$



Press **Y=** and enter the formula above.

```
Plot1 Plot2 Plot3
\Y1=X-(X^3+2X-4)
/(3X^2+2)
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```



Next, press **2ND | TBL SET** and specify **ASK** for the **Indpnt**: variable (x).

```
TABLE SETUP
TblStart=1.5
ΔTbl=.1
Indpnt: Auto ASK
Depend: Auto Ask
```

X	Y1	
X=1.5		



Press **2nd | TABLE**. Since the root is known to be

between 1 and 2, enter 1.5 as the first estimate of the root, x_0 .

Press **ENTER** and the new estimate for the root appears (1.2286); however, it only displays 4 decimal places.

X	Y ₁	
1.5	1.2286	

To see more decimal places for this 1st iteration of Newton's method, move the cursor over the 1.2286 and it expands to 1.22857142857.

X	Y ₁	
1.5	1.2286	
Y ₁ =1.22857142857		

To get an even better estimate of the root with a 2nd iteration of Newton's method, move the cursor down below 1.5 and enter 1.22857142857 (the previous estimate). Then press **ENTER** to obtain the answer 1.18084996159. (You would see the additional decimal places when the cursor is moved over the 1.1808.)

X	Y ₁	
1.5	1.2286	
1.2286	1.1808	
X=1.22857142857		

X	Y ₁	
1.5	1.2286	
1.2286	1.1808	
1.1808	1.1795	
X=1.18084996159		

Now move the cursor below 1.2286 and enter the last estimate (1.18084996159). Press **ENTER** to get the 3rd estimate of the root (1.1795).

X	Y ₁	
1.5	1.2286	
1.2286	1.1808	
1.1808	1.1795	
Y ₁ =1.1795100544		

Move the cursor over 1.1795 to get a final and more accurate answer of 1.1795100544.